

## UNIT - I

### SYNCHRONOUS GENERATOR

- Constructional details
- Types of rotors
- Winding factors
- EMF equation
- Synchronous reactance
- Armature reaction
- Phasor diagrams of non salient pole synchronous generators connected to infinite bus.
- Synchronizing and parallel operation.
- Synchronizing Torque
- Change of excitation and mechanical input.
- Voltage regulation - EMF, MMF, 2 PF and A.S.A methods.
- Steady state power.
- Angle characteristics
- Two reaction theory
- Slip test - short circuit transients.
- Capability curves.

(11)

## CONSTRUCTION OF ALTERNATORS.

An alternator has 3-phase winding on the stator and a d.c field winding on the rotor.

### 1. Stator :

It is stationary part of the machine and is build up of sheet-steel laminations having slots on its inner periphery. A 3 phase winding is placed in these slots and serves as the armature winding of the alternator. The alternator winding is always connected in star and the neutral is connected to ground.

### 2. Rotor :

The rotor carries a field winding which is supplied with direct current through two slip rings by a separated d.c source.

Namely,

1. salient (or) projecting pole type.
2. Non-salient pole (or) cylindrical type.

Salient pole type.



- Used for slow and moderate speed alternators
- least expensive
- Cannot be employed in high speed generators.
- Poles are made of thick steel lamination riveted together and are fixed to rotor by a dove tail joint.
- The pole phases are usually provided with slots for damper windings.
- Damper winding prevent hunting

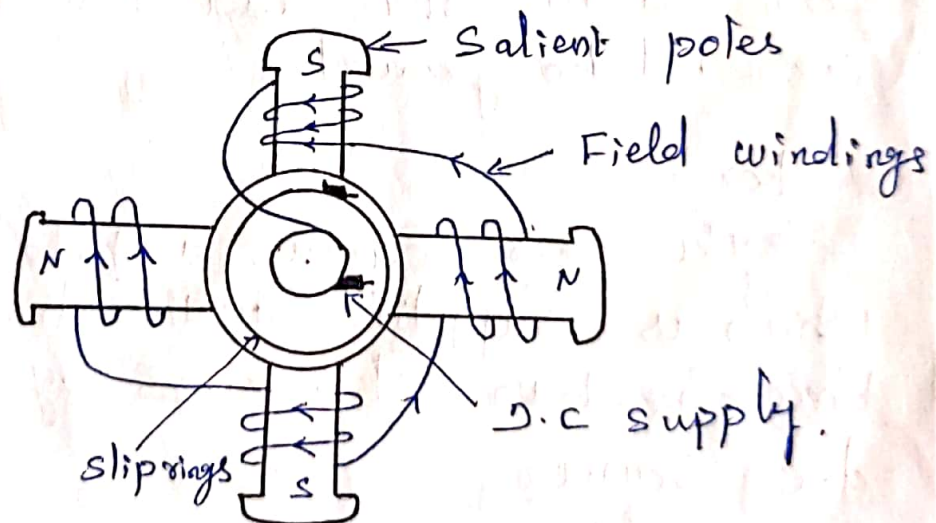


Fig. Salient pole rotor.

- The pole faces are so shaped that the radial air gap length increases from the pole centre to the pole tips. so that flux distribution and generated emf is sinusoidal.

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- The ends of the field windings are connected to a d.c source through slip rings carrying brushes and mounted on the shaft of the field structure.

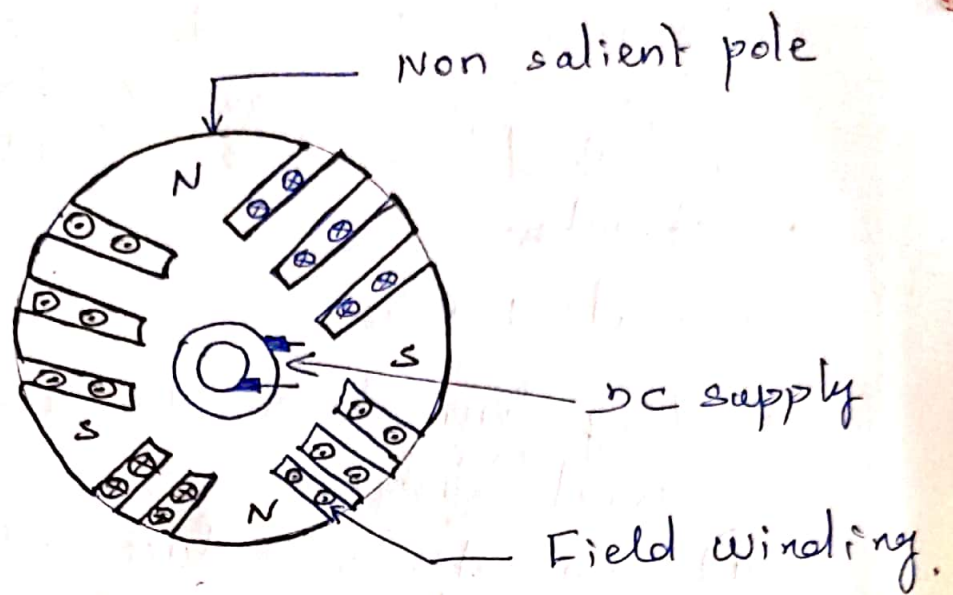
Special features.

- i) They have large diameter and short axial length.
- ii) Laminations reduce eddy current losses.
- iii) Employed in hydraulic turbines and diesel engines.
- iv) speed is from 120 to 400 rpm.

Smooth cylindrical or non-salient pole type.

- Used in very high speed alternators driven by steam turbine.
- To reduce the peripheral velocity, the diameter of the rotor is reduced and axial length is increased.
- It consists of a cylindrical steel forging fabrication.
- The coil ends are fastened by metal rings.
- The regions forming the poles are usually left unslotted as shown in fig.





### Special Features.

- i) small diameter very long axial length.
- ii) Less windage loss
- iii) speed is from 1,000 to 3,000 rpm.

### Working of alternator:

The field magnets are magnetised by applying 125 V or 250 V through slip rings. The rotor and hence the field magnets are driven by the prime mover.

As the rotor rotates, the armature conductors are cut by the magnetic flux. Hence an emf is induced in the armature conductors. The direction of induced emf can be found by Fleming's right hand rule and frequency is given by,



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$$f = \frac{PN}{120}$$

where  $N, P \rightarrow$  speed of rotor in rpm.  
Number of rotor poles.

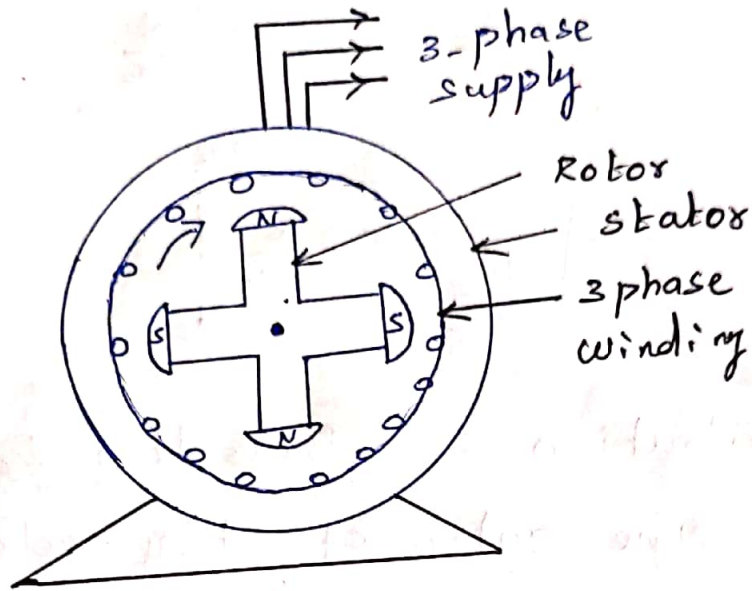
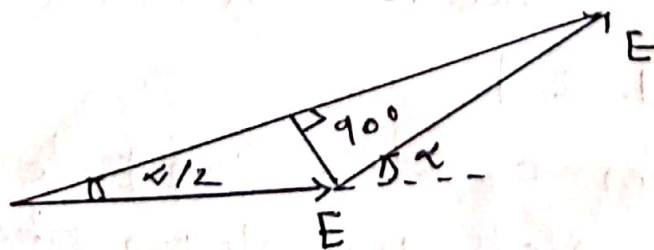


Fig: sectional view of salient pole alternator.

coil span factor or pitch factor

The ratio of vector sum of induced emf's per coil to the arithmetic sum of induced em.f per coil is known as pitch factor ( $k_p$ ) or coil span factor ( $k_c$ ). It is always less than unity.



Resultant induced emf  $E_r = 2E \cos \frac{\alpha}{2}$

$$k_p = \frac{\text{vector sum}}{\text{Arithmetic sum}}$$
$$= \frac{2E \cos \frac{\alpha}{2}}{2E}$$

$$k_p = \cos \frac{\alpha}{2}$$

Distribution or Breadth factor.

The ratio of the vector sum of the emfs induced in all the coils distributed in a number of slots under one pole to the arithmetic sum of the e.m.f.s induced is known as breadth factor or distribution factor.

$$k_d = \frac{(\text{vector sum}) \text{ E.M.F induced in a distributed winding}}{(\text{Arithmetic sum}) \text{ E.M.F induced if the winding would have been concentrated.}}$$

$$k_d < 1$$

Let number of slots per pole =  $n$   
Number of slots per pole per phase =  $m$

(4)

Induced emf in each coil side =  $E_c$

Angular displacement between the

$$\text{slots } \beta = \frac{180^\circ}{n}$$

$$k_d = \frac{\sin m\beta}{\frac{m \sin \beta}{2}}$$

Example: calculate the pitch factor for the under given winding: 36 stator slots, 4 poles, 1 coil span 1 to 8.

solution:-

36 stator slots, 4 pole, coil span 1 to 8.

$$\text{slots/pole} = \frac{36}{4} = 9$$

The coil span falls short by  $\left(\frac{2}{9}\right) \times 180$

$$\alpha = 40^\circ$$

$$\therefore k_p = \cos \frac{\alpha}{2}$$

$$= \cos \frac{40^\circ}{2}$$

$$k_p = 0.94$$



## EMF EQUATION OF AN ALTERNATOR

Let  $Z_{ph}$  = number of conductors or coil sides in series / phase.

$$Z_{ph} = Z T_{ph}$$

$T_{ph}$  = Number of coils or turns per phase

$P$  = number of poles.

$f$  = frequency of induced emf in Hz.

$\phi$  = flux / pole in webers

$$k_d = \frac{\sin \frac{m\beta}{2}}{m \sin \beta/2}$$

$$k_c(\text{or}) k_p = \cos \frac{\alpha}{2}$$

$$k_f = \text{Form factor} \approx \cos \frac{\pi}{2} = 1.11$$

$N$  = rotor speed in rpm.

For one revolution of the rotor each stator conductor is cut by a flux of  $\phi P$  webers.

$$d\phi = \phi P$$

$$dt = 60/N \text{ second.}$$

(5)

Average emf induced per conductor =  $\frac{d\phi}{dt}$

$$= \frac{\phi P}{60/N} = \frac{\phi F N}{60}$$

We know that  $f = \frac{PN}{120}$  or  $N = \frac{120f}{P}$

Substituting this value of  $N$ , we get average emf per conductor,

$$= \frac{\phi P}{60} \times \frac{120f}{P} = 2f\phi \text{ volts.}$$

If there are 2 ph conductors in series/ph then Average emf / phase =  $2f\phi$  2 ph volts.

$$= 4f\phi \text{ Tph volts.}$$

RMS value of emf/phase =  $1.11 \times 4f\phi \text{ Tph}$

$$= 4.44 f\phi \text{ Tph volts.}$$

∴ Actually available voltage / phase

$$= 4.44 k_p k_d f\phi \text{ Tph volts}$$

$k_p k_d = k_w =$  winding factor.

If the alternator is star connected then the line voltage is  $\sqrt{3}$  times the phase voltage.

Example: A 4 pole, 150 Hz star connected alternator has a flux per pole of 0.12 wb. It has 4 slots per pole per phase, conductors per slot being 4. If the winding coil span is  $150^\circ$ , find the emf.

Solution:-

Number of slots/pole/phase  $m = 4$

Number of slots/pole  $n = m \times \text{number of phases}$   
 $= 4 \times 3 = 12$

Number of slots/phase  $= m \times \text{number of poles}$   
 $= 4 \times 4 = 16$

$Z_{ph} = \text{number of conductors} \times \text{number of slots per phase} = 4 \times 16 = 64$

Number of turns per phase  $T_{ph} = \frac{Z_{ph}}{2}$   
 $= \frac{64}{2} = 32$

Angular displacement between the slots

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{12} = 15^\circ$$

Distribution factor  $k_d = \frac{\sin m\beta/2}{m \sin \beta/2}$



(6)

$$= \frac{\sin \frac{4 \times 15^\circ}{2}}{4 \sin \frac{15^\circ}{2}}$$

$$k_{cd} = \frac{\sin 30^\circ}{4 \sin 7.5^\circ} = \frac{0.5}{0.5221} \\ = 0.9576$$

chording angle  $\alpha = 180^\circ - \text{coil span}$   
 $= 180^\circ - 150^\circ = 30^\circ$

pitch factor  $k_{cp} = \cos \frac{\alpha}{2} = \cos \frac{30^\circ}{2}$   
 $= 0.9659$

phase voltage  $E_{ph} = 4.44 f T_{ph} k_p k_{cd}$   
 $= 4.44 \times 50 \times 0.12 \times 32 \times 0.9659 \times 0.9576$

$$E_{ph} = 788.497 \text{ V}$$

Line voltage  $E_L = \sqrt{3} E_{ph}$

$$= \sqrt{3} \times 788.497$$

$$E_L = 1365.718 \text{ V}$$

Example: A 3-phase 4 pole synchronous generator has a double layer winding having four turns per coil placed in a total of 48 slots. If the flux per pole of the generator is  $2 \times 10^6$  lines and speed of the rotor is 1500 rpm, calculate the magnitude of generated voltage per phase.

Given data:

$$P = 4,$$

$$\text{Number of slots} = 48$$

$$\text{No. of turns/coil} = 4$$

$$\phi = 2 \times 10^6 \text{ lines}$$

Soln.:-

$$\text{no. of conductors / slot, } Z = 4 \times 2 = 8$$

$$Z_{ph} = \frac{Z}{P} = 2 \text{ Tph}$$

$$\text{no. of conductors / phase } Z_{ph}$$

$$Z_{ph} = \frac{8 \times 48}{3} = 128$$

$$T_{ph} = \frac{Z_{ph}}{2}$$

$$= \frac{128}{2} = 64$$

$$\text{Flux / pole } \phi = 2 \times 10^6 \text{ lines}$$

We know that

$$1 \text{ flux line} = 10^{-8} \text{ wb.}$$

(4)

$$\phi = 2 \times 10^6 \times 10^{-8} = 2 \times 10^{-2} \text{ wb.}$$

Number of slots / poles / ph.

$$m = \frac{48}{4 \times 3} = 4$$

$$n = 4 \times 3 = 12$$

$$\beta = \frac{180^\circ}{12} = 15^\circ$$

$$\text{Distribution factor } k_d = \frac{\sin \frac{m\beta}{2}}{m \sin \beta/2}$$

$$\text{Supply frequency } f = \frac{NP}{120} = \frac{1500 \times 4}{120} = 50 \text{ Hz.}$$

$$k_d = \frac{\sin \frac{4 \times 15^\circ}{2}}{4 \sin \frac{15^\circ}{2}} = 0.957$$

For Full pitched coil

$$\therefore k_p = 1$$

$$\text{Generated emf } E_{ph} = 4.44 f \phi T_{ph} k_w$$

$$= 4.44 \times 0.957 \times 1 \times 2 \times 10^{-2} \times 50 \times 64$$

$$E_{ph} = 272 \text{ V}$$



Home work.

1. Find the no load phase and line voltages of a star connected 3 $\phi$ , 6 pole alternator which runs at 1200 rpm, having flux per pole of 0.1 wb sinusoidally distributed. Its stator has 54 slots having double layer winding. Each coil has 8 turns and the coil is chorded by 1 slot.

Given data

$P = 6$ ,  $N = 1200$  rpm,  $\phi = 0.1$  wb, slots = 54  
coil has 8 turns.

Soln:-

$$f = \frac{PN}{120} = \frac{6 \times 1200}{120} = 60 \text{ Hz}$$

$$\text{No. of conductors / ph} = \frac{54 \times 8}{3} = 144$$

$$T_{ph} = \frac{217b}{2} = \frac{144}{2} = 72$$

Here, the winding is chorded by one slot it is short pitched by  $\frac{1}{9}$  or  $180^\circ / 9 = 20^\circ$

$$k_p = \cos \frac{\alpha}{2} = \cos \frac{20}{2} = 0.98$$

$$\text{number of slots / pole} = \frac{54}{6} = 9$$

(8)

slot angle  $\beta = \frac{180^\circ}{n} = \frac{180^\circ}{9} = 20^\circ$

$$m = \frac{54}{6} = 9$$

$$k_d = \frac{\sin \frac{3 \times 20}{2}}{3 \sin \frac{20}{2}} = 0.96$$

$$E_{ph} = 4.44 f T_{ph} k_p k_d$$
$$= 4.44 \times 60 \times 0.1 \times 72 \times 0.98 \times 0.96$$

$$E_{ph} = 1805 \text{ V}$$

$$E_L = \sqrt{3} E_{ph}$$

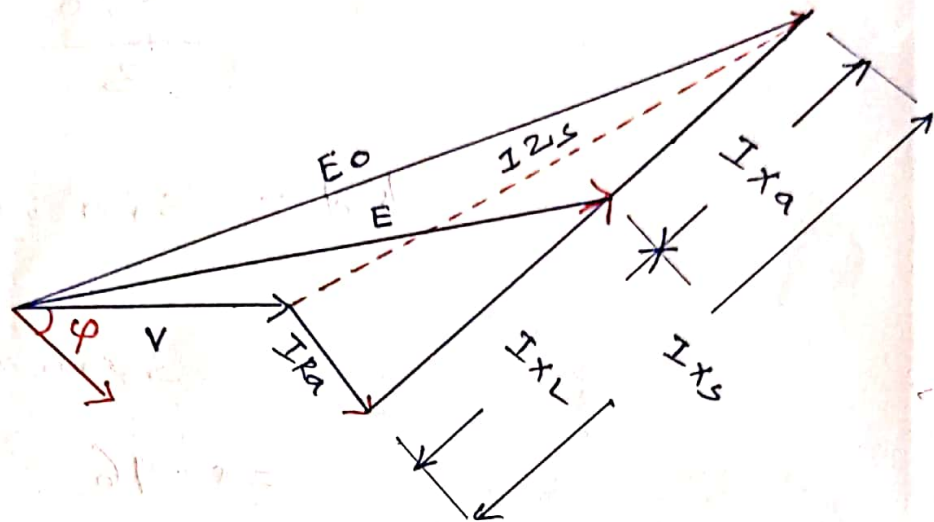
$$= \sqrt{3} \times 1805$$

$$E_L = 3125 \text{ V}$$

SYNCHRONOUS REACTANCE ..

For the same field excitation, terminal voltage is decreased from its no load value  $E_0$  to  $V$ . This is because of

1. drop due to armature resistance,  $I R_a$
2. drop due to leakage reactance,  $I X_L$
3. drop due to armature reaction.



$I X_a$  represents the voltage drop due to armature reaction.

The leakage reactance  $X_L$  and the armature reaction  $X_a$  may be combined to give synchronous reactance  $X_s$ .

$$\text{Hence } X_s = X_L + X_a$$

Total voltage drop in an alternator

$$= I R_a + j I X_s$$

$$= I Z_s$$

$Z_s \rightarrow$  synchronous impedance



(9)

Example: A 3 $\phi$ , 8 pole alternator has a star connected winding with 120 slots and 12 conductors per slot. Flux per pole is 0.04 wb and is sinusoidally distributed. Machine runs at 750 rpm. Find the frequency, phase and line emf, if pitch factor for third harmonic is 0.809.

Solution:-

$$P = 8 \quad S = 120 \quad 2/S = 12$$

$$\phi = 0.04 \text{ wb} \quad N_s = 750 \text{ rpm} \quad k_p = 0.809$$

Emf induced in the alternator is given by,

$$E_{ph} = 4.44 f \cdot k_b k_p N_{ph} \phi$$

$$\text{No. of conductors / slots} = 12$$

$$\begin{aligned} \text{Total no. of turns} &= \frac{12 \times S}{2} \\ &= \frac{12 \times 120}{2} \\ &= 720 \end{aligned}$$

$$\begin{aligned} \text{No. of turns / phase} \quad N_{ph} &= \frac{720}{3} = 240 \\ &= \frac{\pi P}{S} \end{aligned}$$

$$N_s = 120 f / P$$

$$P = 120 f / N_s$$

$$P = 8$$

$$f = 50$$

$$\therefore \gamma = \frac{\pi P}{S} = \frac{180 \times 8}{120}$$

$$\gamma = 12^\circ$$

$$m = \frac{S}{Pq} = 5$$

$$k_b = \frac{\sin m\gamma/2}{m \sin \gamma/2}$$

$$k_b = \frac{\sin (5 \times 12) / 2}{5 \sin (12/2)} = 0.9567$$

$$k_b = 0.9567$$

$$\text{Emf generated / phase} = 4.44 f k_b k_p N \phi$$

$$= 4.44 \times 50 \times 0.9567 \times 0.809 \times 240 \times 0.04$$

$$E_{ph} = 1649.4855 \text{ Volts.}$$

$$\text{Line voltage} = \sqrt{3} E_{ph} = 2.857 \text{ kV.}$$

## ARMATURE REACTION IN ALTERNATOR.

The effect of armature flux on the flux produced by field ampere-turns is called armature reaction.

Two things are worth nothing about the armature reaction in an alternator.

① The armature flux and the flux produced by rotor ampere-turns rotate at the same speed in the same direction and, therefore, the two fluxes are fixed in space relative to each other.

② The modification of flux in the air gap due to armature flux depends on the magnitude of stator current and on the power factor of the load. It is the load power factor which determines whether the armature flux distorts, opposes or helps the flux produced by rotor ampere-turns.

The following three cases:



- (i) when load p.f is unity
- (ii) when load p.f is zero lagging
- (iii) when load p.f is zero leading.

(i) when load p.f is unity :

Fig. shows an elementary alternator on no-load. Since the armature is on open ckt, there is no stator current and the flux due to rotor current is distributed symmetrically in the air-gap. No armature flux is produced since no current flows in the armature winding.

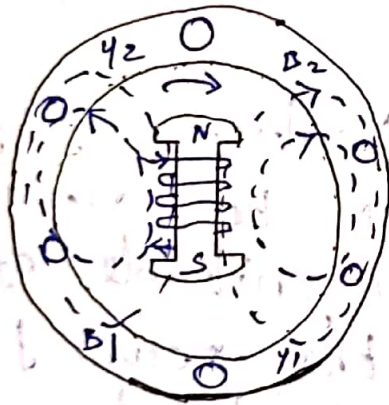


Fig: Armature is on open circuit

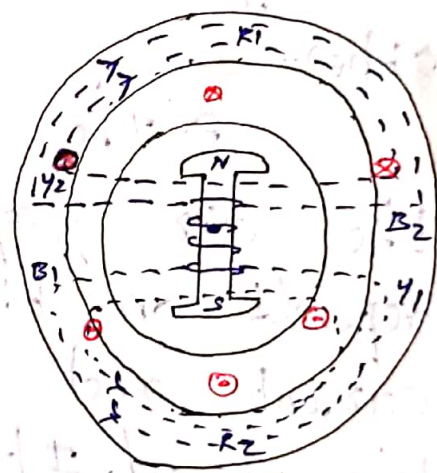
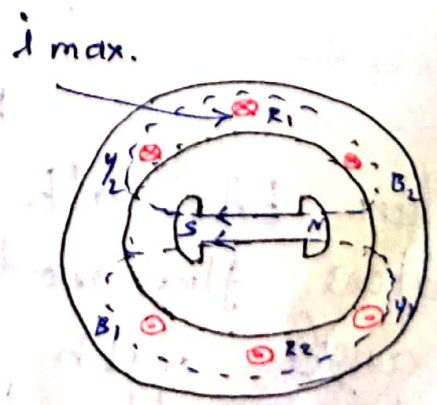
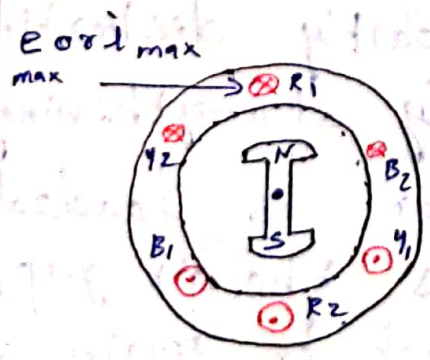


Fig. When a resistive load.

when a resistive load is connected across the terminals of the alternator. The armature flux is clockwise due to currents in the top conductors and anticlockwise due to currents in the bottom

conductors. In this case, the flux in the air-gap is distorted but not weakened. Therefore, at unity p.f. the effect of armature reaction is merely to distort the main field; there is no weakening of the main field and the average flux practically remains the same.

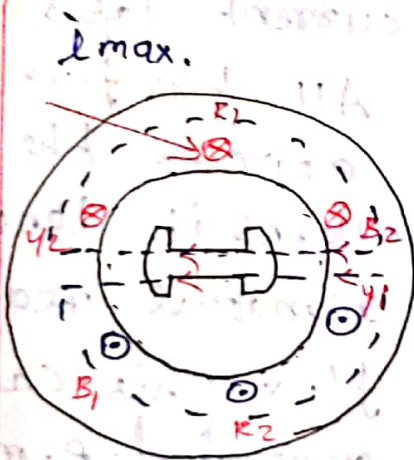
(ii) When load p.f. is zero lagging. When a pure inductive load is connected across the terminals of the alternator, current lags behind the voltage by  $90^\circ$ . All the flux produced by armature current opposes the field flux and therefore, weakens it, hence at zero p.f. lagging, the armature reaction weakens the main flux. This causes a reduction in the generated e.m.f.





(iii) When load p.f. is zero leading.  
 When a pure capacitive load is connected across the terminals of the alternator, the current in the armature winding will lead the induced e.m.f. by  $90^\circ$ . Obviously, the effect of armature reaction will be the reverse that for pure inductive load.

At zero p.f. leading, the armature leading reaction strengthens the main flux.



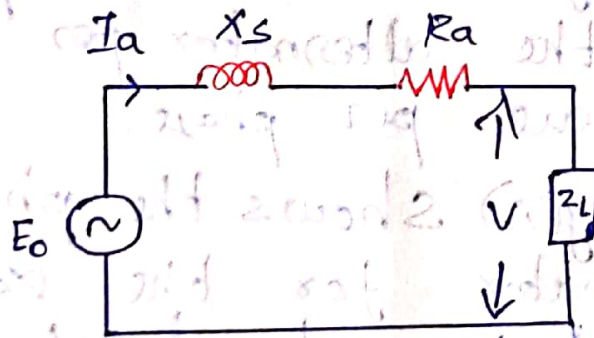
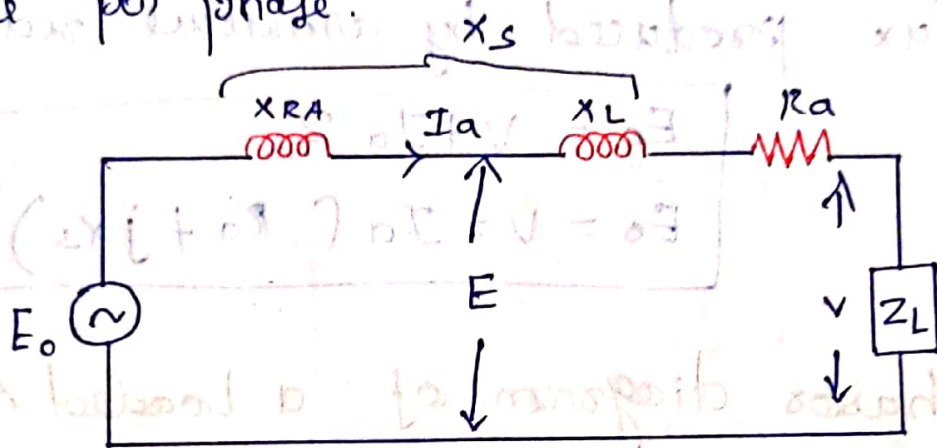
For intermediate values of p.f., the effect of armature reaction is partly distorting and partly weakening for inductive loads. For capacitive loads, the effect of armature reaction is partly distorting and partly strengthening.

When the alternator is loaded, the armature flux modifies the air-gap flux. Its angle with respect to main flux depends on the load p.f.



### SYNCHRONOUS REACTANCE.

The sum of armature leakage reactance ( $X_L$ ) and reactance of armature reaction ( $X_{AR}$ ) is called synchronous reactance  $X_s$ . Note that all quantities are per phase.



The synchronous reactance is a fictitious reactance employed to account for the voltage

effects in the armature circuit produced by the actual armature leakage reactance and the change in the air gap flux caused by armature reaction. The circuit then reduces to the one shown in fig.

$$\text{Synchronous impedance, } Z_s = R_a + jX_s$$

The synchronous impedance is the fictitious impedance employed to account for the voltage effects on the armature circuit produced by the actual armature resistance, the actual armature leakage reactance and the change in the airgap flux produced by armature reaction.

$$E_0 = V + I_a \cdot Z_s$$

$$E_0 = V + I_a (R_a + jX_s)$$

Phasor diagram of a loaded Alternator.

Fig(i) shows the equivalent circuit of the alternator per phase. All quantities are per phase.

Fig(ii) shows the phasor diagram of an alternator for the usual case of inductive load. The armature current  $I_a$  lags the terminal voltage  $V$  by p.f. angle  $\phi$ . The phasor sum of  $V$  and drops  $I_a \cdot R_a$  and  $I_a \cdot X_L$  gives the load induced voltage  $E$ . It is the induced emf after allowing for armature reaction. The phasor sum of  $E$  and  $I_a \cdot X_{AR}$  gives



The no load e.m.f  $E_0$ . Note that in drawing the phasor diagram either the terminal voltage ( $V$ ) or armature current ( $I_a$ ) may be taken as the reference phasor.

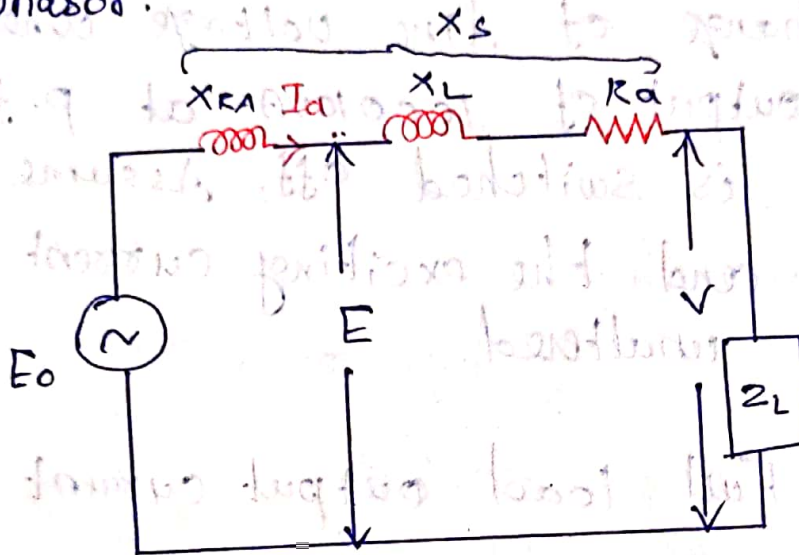


Fig. (i)

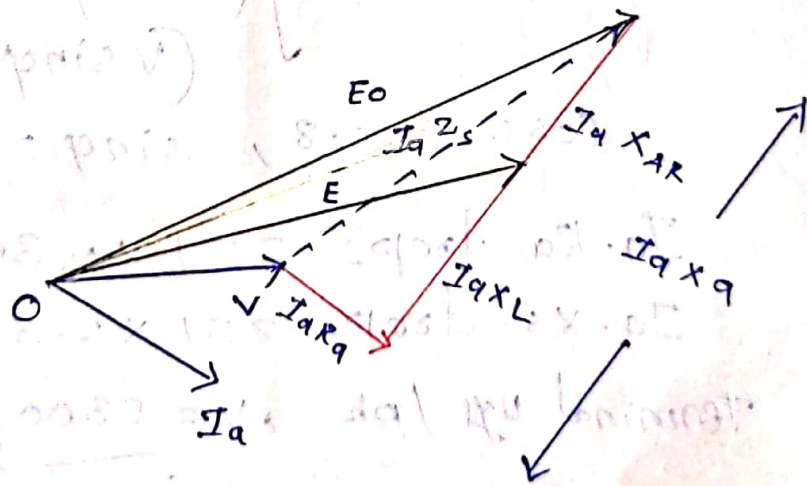


Fig (ii)



Change in line voltage =  $3441 - 2300 = 1141$  volts.

Ex: A 1000 kVA, 2300V, 3-phase, star connected alternator has a resistance of  $0.309 \Omega / \text{ph}$  and a synchronous reactance of  $3.31 \Omega / \text{ph}$ . Calculate the change of line voltage when the rated output of 1000 kVA at p.f of 0.8 lagging is switched off. Assume the speed and the exciting current to remain unaltered.

Soln: Full load output current

$$I_a = \frac{1000 \times 10^3}{\sqrt{3} \times 2300} = 251 \text{ A.}$$

$$E_0 = \sqrt{(V \cos \phi + I_a \cdot R_a)^2 + (V \sin \phi + I_a \cdot X_s)^2}$$

$$\cos \phi = 0.8, \quad \sin \phi = 0.6$$

$$I_a \cdot R_a \text{ drop} = 251 \times 0.309 = 77.6 \text{ volts}$$

$$I_a \cdot X_s \text{ drop} = 251 \times 3.31 = 831 \text{ volts.}$$

$$\text{Terminal vge / ph, } V = 2300 / \sqrt{3} = 1328 \text{ volts.}$$

$$E_0 = \sqrt{(1328 \times 0.8 + 77.6)^2 + (1328 \times 0.6 + 831)^2}$$
$$= 1987 \text{ volts.}$$

$$\text{Line value} = \sqrt{3} E_0 = 3441 \text{ volts.}$$

# VECTOR DIAGRAMS OF A LOADED ALTERNATOR.

Let  $E_0$  = No load e.m.f, it represents the maximum value of the induced e.m.f.

$E$  = Loaded induced e.m.f. It is the induced e.m.f after allowing for armature reaction.

$E$  is vectorially less than  $E_0$  by  $I_x a$ .

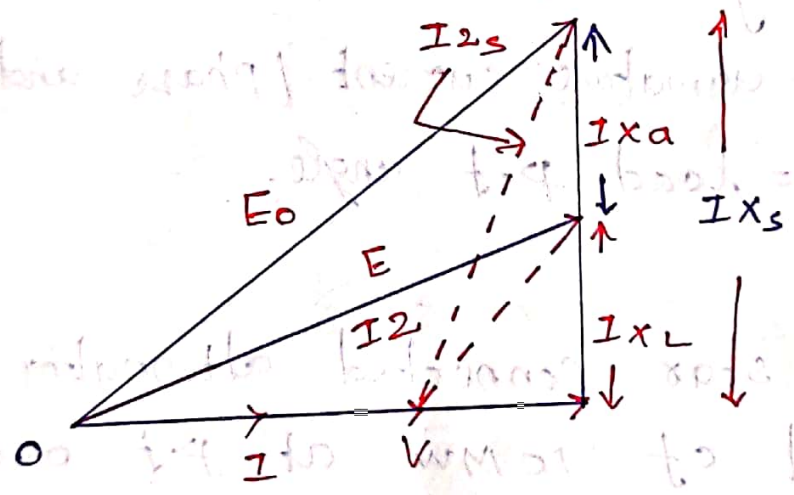


Fig.(a) Unity p.f

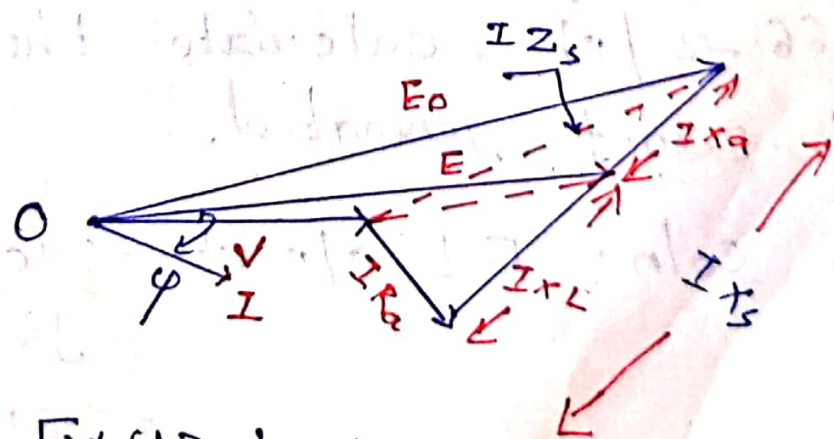


Fig.(b) Lagging p.f

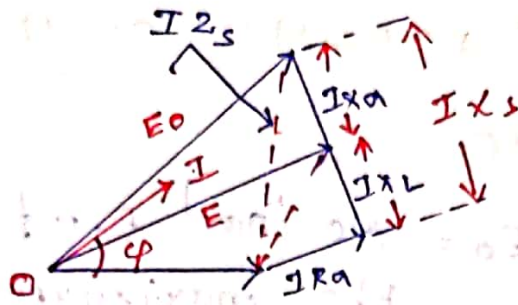


Fig. (c) Leading p.f

$V$  = Terminal voltage, it is vectorially less than  $E_0$  by  $I Z_s$

$$Z_s = \sqrt{R_a^2 + X_L^2}$$

$I$  = armature current / phase and

$\varphi$  = load p.f angle.

Ex: A 3 $\phi$ , star connected alternator supplies a load of 10 MW at p.f 0.85 lagging and at 11 kV (terminal vge). Its resistance is 0.1  $\Omega$ /ph and synchronous reactance 0.66  $\Omega$ /ph. calculate the line value of e.m.f generated.

$$\begin{aligned} \text{Soln: F.L o/p ct} &= \frac{10 \times 10^6}{\sqrt{3} \times 11,000 \times 0.85} \\ &= 618 \text{ A} \end{aligned}$$



$$I R_a \text{ drop} = 618 \times 0.1 = 61.8 \text{ V}$$

$$I X_s \text{ drop} = 618 \times 0.66 = 408 \text{ V}$$

$$\text{Terminal vge /ph} = 11,000 / \sqrt{3} = 6350 \text{ V}$$

$$\phi = \cos^{-1}(0.85) = 31.8^\circ$$

$$\sin \phi = 0.527$$

$$E_0 = \sqrt{(V \cos \phi + I R_a)^2 + (V \sin \phi + I X_s)^2}$$

$$= \sqrt{(6350 \times 0.85 + 61.8)^2 + (6350 \times 0.527 + 408)^2}$$

$$= 6625 \text{ V}$$

$$\text{Line emf} = \sqrt{3} \times 6625 = 11,486 \text{ volts}$$

### VOLTA GE REGULATION

It is clear that with change in load, there is a change in terminal voltage of an alternator. The magnitude of this change depends not only on the load but also on the load p.f.

The voltage regulation of an alternator is defined as the rise in voltage when full load is removed, divided by the rated terminal voltage.

$$\therefore \text{regulation} = \frac{E_0 - V}{V} \times 100.$$

(i) In the case of leading load, p.f. terminal  $V_{ge}$  will fall on removing the full load. Hence, regulation is negative in that case.

(ii) The rise in  $V_{ge}$  when full load is thrown off is not the same as the fall in voltage when full load is applied.

Determination of voltage regulation.

In the case of small machines, the regulation may be found by direct loading.

In the case of large machines, the cost of finding the regulation by direct loading becomes prohibitive. Hence other indirect methods are used.



1. Synchronous Impedance method
2. The ampere turns or M.M.F method
3. zero p.f. or potier method.

All the methods require.

(i) Value of  $R_a$

Armature resistance  $R_a$  per phase can be measured directly by voltmeter and ammeter method or by using wheatstone bridge. Generally a value 1.6 times the d.c. value is taken.

(ii) Open circuit characteristics.

As in d.c. machine, this is plotted by running the machine on no load and by noting the values of induced voltage and field excitation current. It is just like the B-H curve.

(iii) Short circuit characteristics.

It is obtained by short circuiting the armature windings through a low resistance ammeter. During this test, the speed which is not necessarily synchronous, is kept constant.



## SYNCHRONOUS IMPEDANCE METHOD

Following steps are involved in this method.

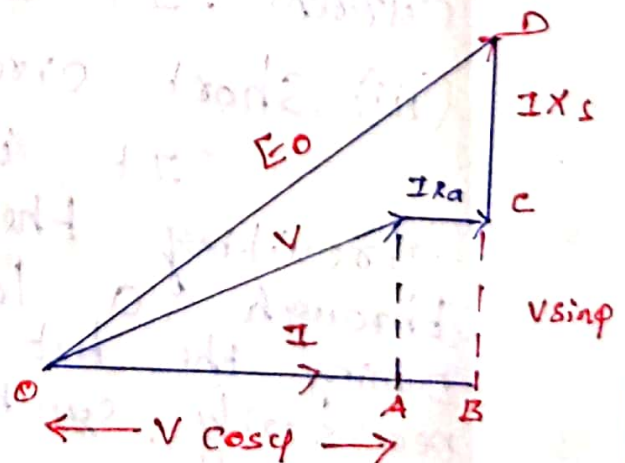
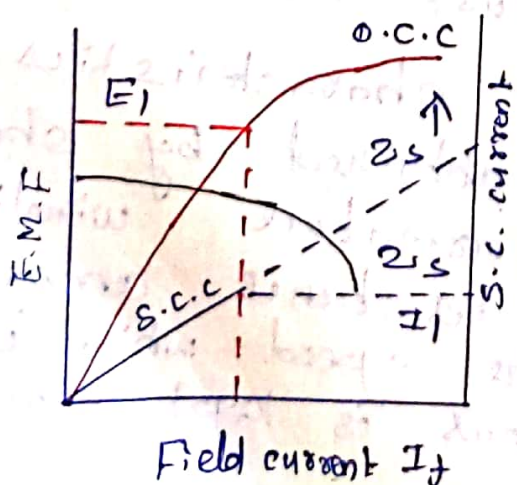
1. O.C.C is plotted from the obtained data.
2. Similarly, S.C.C. is drawn from the data given by the short circuit test. It is a straight line passing through the origin. Both these curves are drawn on a common field current base.

$$\therefore E_1 = I_1 Z_s, \therefore Z_s = \frac{E_1 \text{ (open ckt)}}{I_1 \text{ (short ckt)}}$$

3. Since  $R_a$  can be found,

$$X_s = \sqrt{Z_s^2 - R_a^2}$$

4. Knowing  $R_a$  and  $X_s$ , vector diagram as shown in fig.



Here  $OD = E_0$ ,  $\therefore E_0 = \sqrt{OB^2 + BD^2}$

$$E_0 = \sqrt{(V \cos \phi + I R_a)^2 + (V \sin \phi + I X_s)^2}$$

$$\therefore \gamma_{reg} = \frac{E_0 - V}{V} \times 100$$

Ex: From the following test results, determine the voltage regulation of a 2000 V, 1  $\phi$  alternator delivering a current of 100 A at (i) unity p.f (ii) 0.8 leading p.f and (iii) 0.71 lagging p.f.

Test results: Full load current of 100 A is produced on short circuit by a field excitation of 2.5 A, an emf of 500 V is produced on open circuit by the same excitation. The armature resistance is 0.8  $\Omega$ .

Soln:-  $Z_s = \frac{\text{o.c volts}}{\text{s.c current}} = \frac{500}{2.5} = 200 \Omega$

For same excitation =  $500/100 = 5 \Omega$

$$X_s = \sqrt{Z_s^2 - R_a^2}$$

$$= \sqrt{5^2 - 0.8^2} = 4.936 \Omega$$



(i) Unity p.f  $I_a R_a = 100 \times 0.8 = 80 \text{ V}$

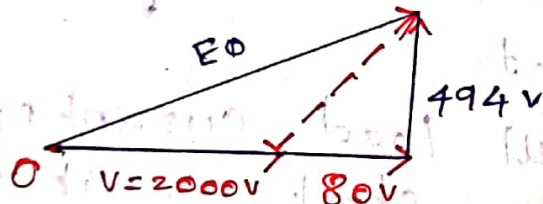
$I X_s = 100 \times 4.936 = 494 \text{ V}$

$E_0 = \sqrt{(2000 + 80)^2 + 494^2}$

$E_0 = 2140 \text{ V}$

$\% \text{ reg} = \frac{2140 - 2000}{2000} \times 100$

$= 7 \%$



(a) Unity p.f.

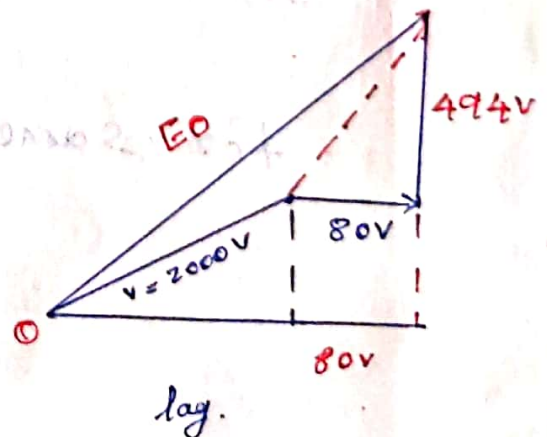
(ii) p.f = 0.8 (lead)

$E_0 = \sqrt{(2000 \times 0.8 + 80)^2 + (2000 \times 0.71 + 494)^2}$

$= 2432 \text{ V}$

$\% \text{ reg} = \frac{2432 - 2000}{2000} \times 100$

$= -9 \%$

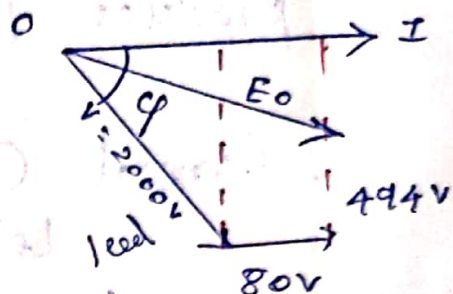




(iii)  $P.f = 0.71 (\text{lag})$

$$E_o = \sqrt{(2000 \times 0.71 + 80)^2 + (2000 \times 0.71 + 494)^2}$$
$$= 2432 \text{ V}$$

$$\% \text{ reg} = \frac{2432 - 2000}{2000} \times 100$$
$$= 21.6\%$$



### ROTHERT'S M.M.F OR AMPERE-TURNS METHOD.

This method also utilizes O.C and S.C data, but is the converse of the E.M.F method in the sense that armature leakage reactance is treated as an addition -al armature reaction.

Now, field A.T required to produce a voltage of V on full load is the vector sum of the following.

(i) Field A.T required to produce a vge of V on <sup>no</sup> full load. This can be found from O.C.C

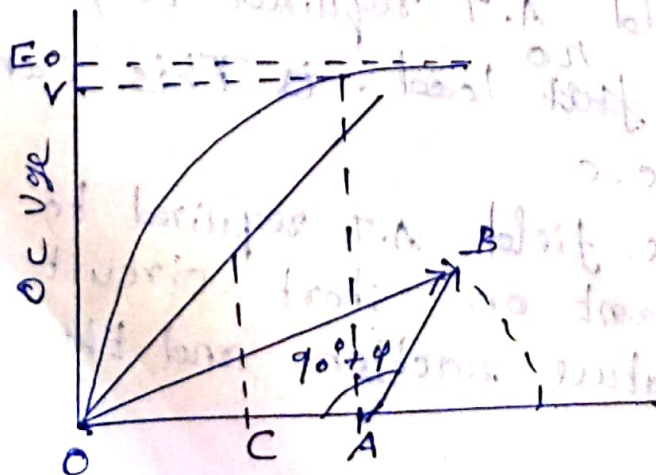
(ii) The field A.T required to produce full load current on short circuit balances the armature reaction and the impedance drop.

In other words, the demagnetising armature A.T on full load are equal and opposite to the field A.T required to produce full load current on short circuit.

General Case :

Let us consider the general case when the p.f has any value between zero (leading or lagging) and unity.

Field ampere turns OA corresponding to  $V$  (or  $V + IR_a \cos \phi$ ) is laid off horizontally. Then  $AB_1$ , representing full load short circuit field A.T is drawn at an angle of  $(90^\circ + \phi)$  for a lagging p.f. The total field AT are given by  $OB_1$ , for a leading p.f short circuit A.T =  $AB_2$  is drawn at an angle of  $(90^\circ - \phi)$  and for unity p.f,  $AB_3$  is drawn at right angles as shown.



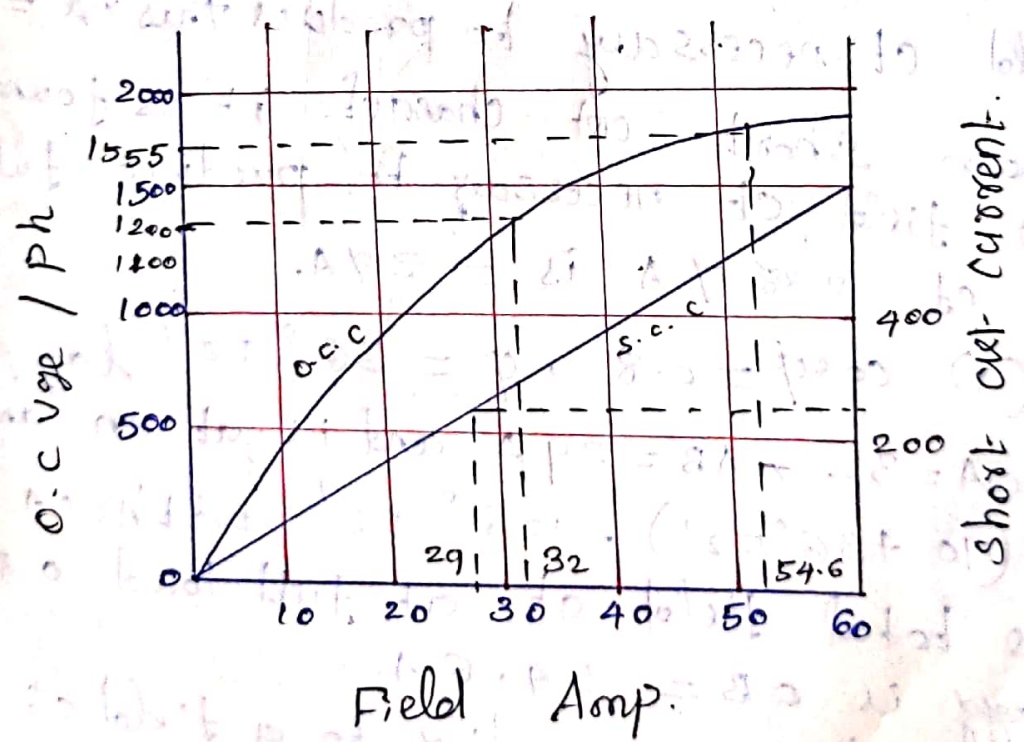
$$\% \text{ Reg} = \frac{E_0 - V}{V} \times 100$$



Ex: The open ckt and short ckt test readings for a 3 $\phi$  1 star connected, 1000 kVA, 12000V, 50Hz synchronous generator are:

Field Amps:	10	20	25	30	40	50
O.c terminal V:	800	1500	1760	2000	2350	2600
S.c armature current in A:	-	200	250	800	-	-

The armature effective resistance is 0.2  $\Omega$  per phase. Draw the characteristics curve and estimate the full load percentage regulation at (a) 0.8 p.f lagging (b) 0.8 p.f leading.





Soln: The phase vgs are: 462, 866, 1016, 1155, 1357, 1502

$$\text{Full load phase voltage} = \frac{2000}{\sqrt{3}} = 1155 \text{ V}$$

$$\text{Full load current} = \frac{1000,000}{\sqrt{3} \times 2000} = 288.7 \text{ A.}$$

V<sub>gp</sub> /ph at full load at 0.8 pf

$$= V + I R_a \cos \phi$$

$$= 1155 + (288.7 \times 0.2 \times 0.8)$$

$$= 1200 \text{ Volts.}$$

From open cut curve, it is found that field ct necessary to produce this v<sub>gp</sub> = 32 A.

From short cut charact. it is found that the field ct necessary to produce full load ct of 288.7 A is = 29 A.

$$(a) \cos \phi = 0.8, \phi = 36^\circ 52' \text{ lag.}$$

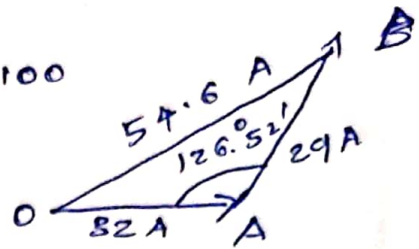
OA = 32, AB = 29 A and is at an angle of  $(90 + 36^\circ 52') = 126^\circ 52'$  with OA.

The total field ct at full load 0.8 pf lag is OB = 54.6 A.

O.c volt corresponding to a field ct of 54.6 A is = 1555 V.

$$\% \text{ reg} = \frac{1555 - 1155}{1155} \times 100$$

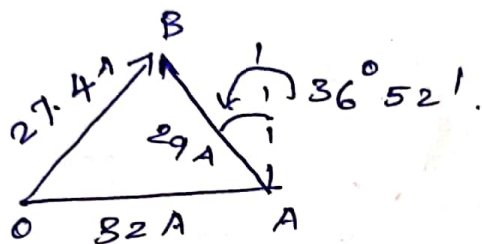
$$= 34.6\%$$



(b) as p.f. is leading

AB is drawn with OA at an angle of

$$90^\circ - 36^\circ 52' = 53^\circ 8' \quad OB = 27.4A$$



O.C vge corresponding to 27.4 A of field excitation is 1080V.

$$\% \text{ reg} = \frac{1080 - 1155}{1155} \times 100$$

$$= -6.4\%$$



## ZERO POWER FACTOR METHOD OR POTIER METHOD.

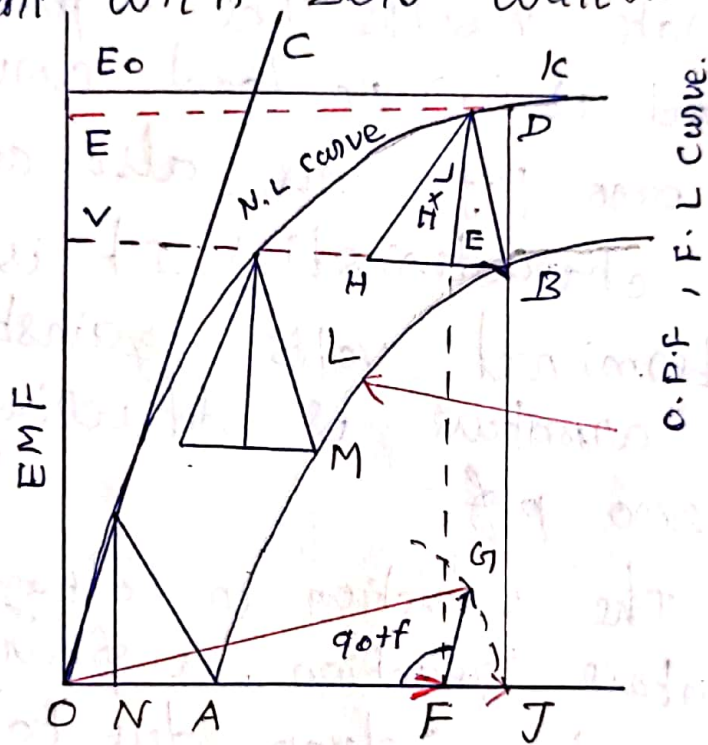
This method is based on the separation of armature-leakage reactance drop and the armature reaction effects. Hence, it gives more accurate results. The experimental data required is (i) no load curve (ii) Full load zero p.f. curve also called wattless load characteristics. It is the curve of terminal volts against excitation when armature is delivering F.L. current at zero p.f.

The reduction in voltage due to armature reaction is found from above and voltage drop due to armature leakage reactance  $x_L$  is found from both. By combining these two,  $E_0$  can be calculated.

The zero p.f. lagging curve can be obtained.

(a) If a similar machine is available which may be driven at no load as a synchronous motor at practically zero p.f.

- (or)  
 (b) by loading the alternator with pure reactors.  
 (c) by connecting the alternator to a 3φ line with ammeters and wattmeter connected for measuring current and power and by so adjusting the field current that we get full load armature current with zero wattmeter reading.



Point B (Fig.) was obtained in this manner when wattmeter was reading zero. Point A is obtained from a short circuit test with full load armature current. Hence,  $OA$  represents field current which is equal and opposite to the demagnetising armature reaction and for balancing leakage reactance drop at full load.

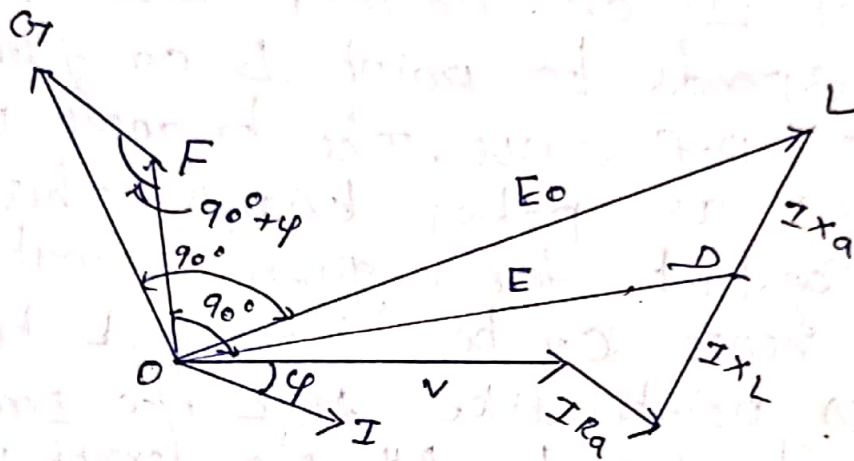


2

From B, BH is drawn equal to and parallel to OA. From H, HD is drawn parallel to initial straight part on N-L curve i.e. parallel to OC, which is tangential to N-L curve. Hence, we get point D on no load curve, which corresponds to point B on full load zero p.f. curve. The triangle BHD is known as potier triangle. This triangle is constant for a given armature current and hence can be transferred to give us other points like M, L etc. Draw DE perpendicular to BH. The length DE represents the drop in voltage due to armature leakage reactance  $X_L$  load and  $I_a R_a$  for  $I_a$ . BE gives field current necessary to overcome demagnetising effect of armature reaction at full load and EH for balancing the armature leakage reactance drop DE.

Let  $V$  be the terminal voltage on full load, then if we add to it vectorially the voltage drop due to armature leakage reactance alone (neglecting  $R_a$ ) then we get voltage  $E = DF$  (and not  $E_0$ ). Obviously, field excitation corresponding to  $E$  is given by OF.  $NA (= BE)$  represents the

field current needed to overcome armature reaction. Hence, if we add NA vectorially to (OF) we get excitation for  $E_0$  whose value can be read from V-L curve.



In fig(i)  $F_0 (= NA)$  is drawn at angle of  $(90^\circ + \phi)$  for a lagging P.f (or it is drawn at an angle of  $90^\circ - \phi$  for a leading p.f). The voltage corresponding to this excitation is  $JK = E_0$ .

$$\therefore \% \text{ reg} = \frac{E_0 - V}{V} \times 100$$

The vector diagram is also shown separately in fig (ii)

Procedure for Potier Method.

1. Suppose we are given  $V$  - the terminal voltage / phase.



(3)

2. Calculate armature leakage reactance,  $X_L$  and hence can calculate  $I X_L$ .
3. Adding  $I X_L$  vectorially to  $V$ , we get voltage  $E$ .
4. From  $N-L$  curve, field excitation for voltage  $E$ . Let it be  $i_{f1}$ .
5. Further, field current  $i_{f2}$  necessary for balancing armature reaction is found from potier triangle.
6. combined  $i_{f1}$  and  $i_{f2}$  vectorially.
7. Read from  $N-L$  curve, the emf corresponding to  $i_f$ , this gives us  $E_0$ . Hence regulation can be found.

## PARALLEL OPERATION OF ALTERNATORS.

The operation of connecting an alternator in parallel with another alternator or with common bus-bars is known as synchronizing.

Need for parallel operation:

- i) the total load requirement cannot be met by a single alternator.
- ii) Increase reliability of electric supply.
- iii) If alternators are operating in parallel, one or more of them can be shut down for preventive maintenance in turn.
- iv) Economical operating cost, the less efficient machines can be shut down when the load requirement is less.

Conditions for parallel operation:-

- i) the terminal voltage of the incoming alternator must be same as that of busbars.
- ii) The frequency of incoming alternator must be same as that of busbars.
- iii) the phase of the incoming machine voltage must be same as that of bus bar voltage relative to the load.
- iv) the phase sequence of the voltage of incoming alternator must be same as that of bus-bar voltage.

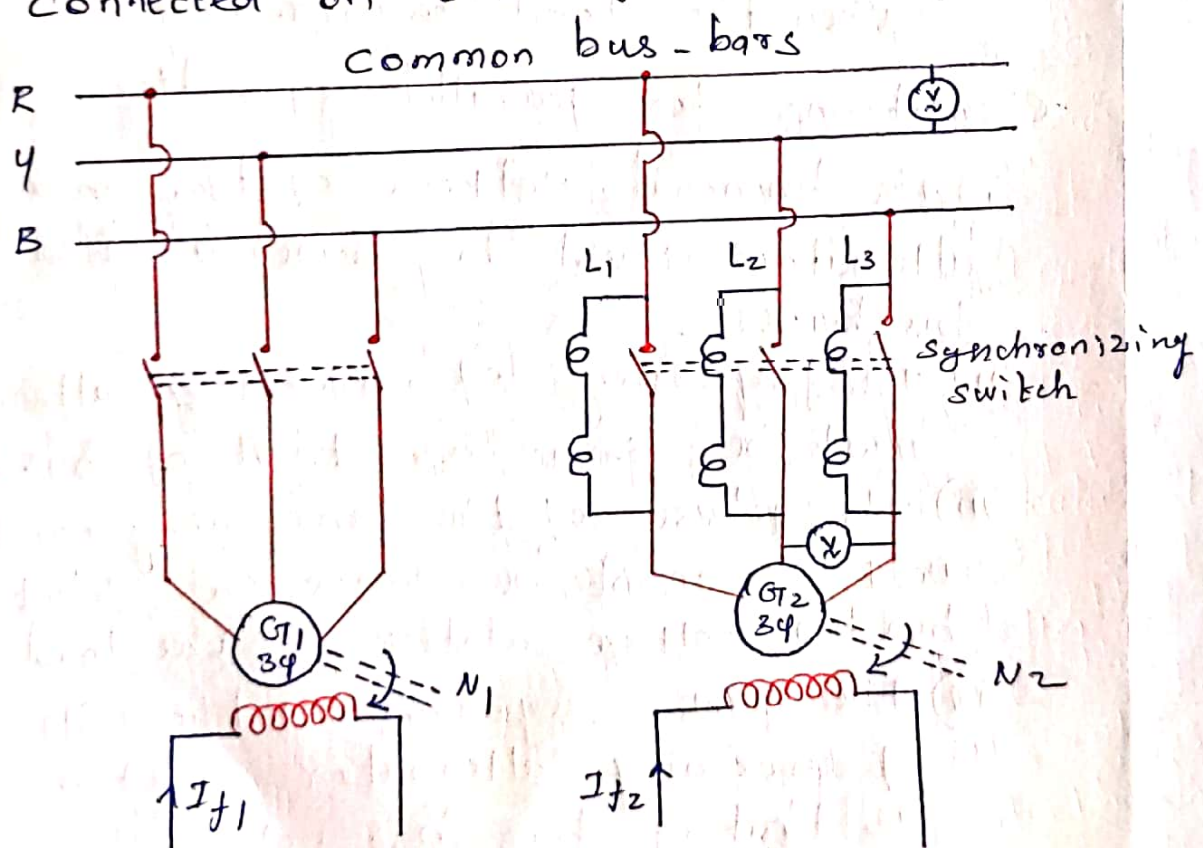


## Synchronizing Procedure.

1. Three dark lamps method
2. Two bright and one dark lamp method
3. Using synchroscope.

### Three dark lamp method.

In figure the alternator  $G_1$  is the already existing one which is connected to the common bus bars RYB.  $G_1$  supplies power to the load. The alternator  $G_2$  is the new alternator which is to be connected to the common bus-bars through the synchronizing switch. Lamps  $L_1, L_2$ , and  $L_3$  connected on the synchronizing switch.



$\omega_2$  speed very close to the synchronizing speed decided by the bus-bar frequency and the number of poles in alternator  $\omega_2$ .

→ If the lamps glow bright and dark in unison it is an indication of the correctness of the phase sequence.

→ The rate of flickering accounts for the frequency difference between the bus bar voltage and the incoming alternator voltage.

Flickering reduced by adjusting the speed of the alternator by its prime mover control.

→ When all the three sets of lamps become dark, the synchronizing switch can be closed and thus the alternator  $\omega_2$  gets synchronised with alternator  $\omega_1$ .

Two Bright and One dark Lamp Method.

→ This method again makes use of three sets of lamps and is useful in finding whether the incoming alternator frequency is higher or lower than the bus bar frequency. But the correctness of the phase sequence cannot be checked by this method.

→ The three set of lamps  $L_1$ ,  $L_2$  and  $L_3$  glow bright and dark one after the other. The sequence in which the three sets of lamps become bright and dark indicates whether the incoming generator frequency is higher or lower than the bus-bar



frequency.

sequence  $L_1, L_2$  and  $L_3 \rightarrow$  Incoming generator frequency  $>$  Bus bar frequency.

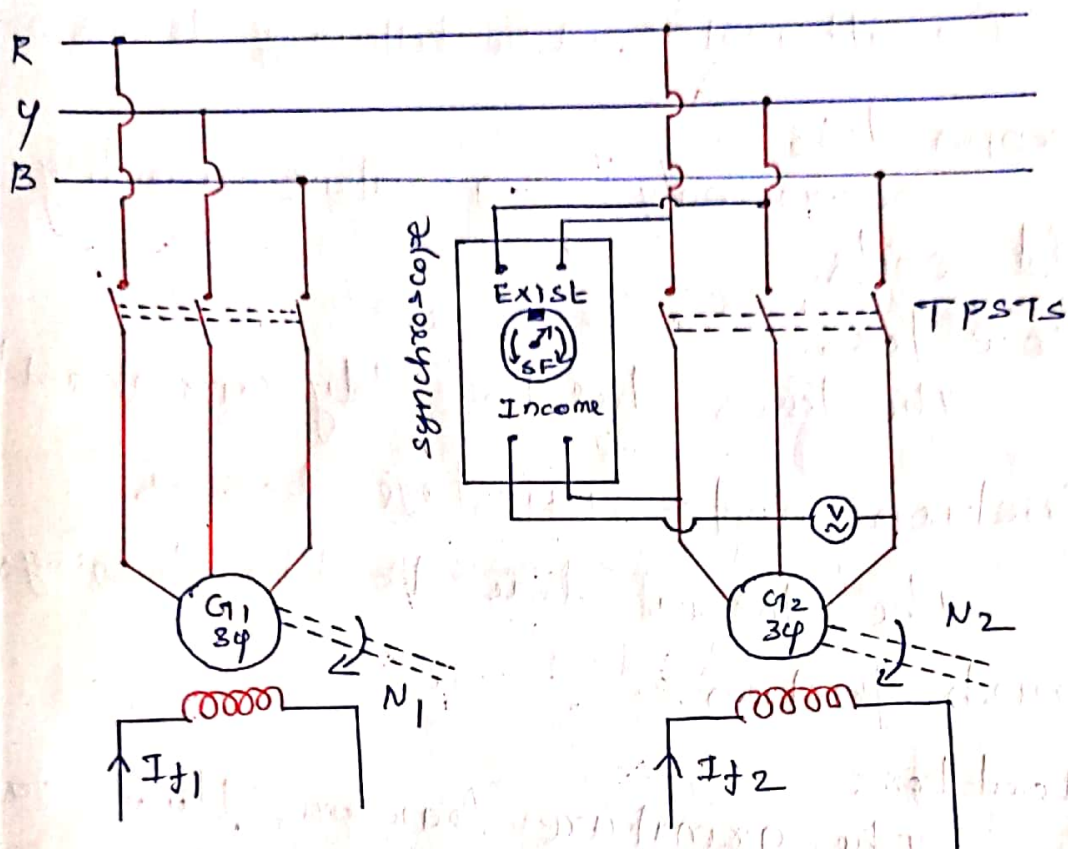
sequence  $L_1, L_3$  and  $L_2 \rightarrow$  Incoming generator frequency  $<$  Bus bar frequency.

After bringing down the rate of flickering to as small as possible, the synchronizing switch is to be closed at the instant when lamp  $L_2$  is dark and lamps  $L_1$  and  $L_3$  are equally bright.

Using Synchroscope  
The best method of synchronizing alternators is by means of a single phase device known as synchroscope. Which provides a more accurate indication of synchronism than do lamps.

The synchroscope is an instrument for indicating difference of phase and frequency between two voltages.

The synchroscope has a circular dial in which a thick line is marked at the top, a clockwise symbol with letter 'F' on one side and an anticlockwise symbol with letter 'S' marked on the other side.



→ The pointer is capable of rotating in both directions. The rate of rotation of the pointer indicates the amount of frequency difference between the alternators.

→ Suitable correction is then made in the speed of the alternator and the rate of rotation is reduced to the smallest possible value.

→ The TPST switch is closed to synchronize the incoming alternator when the pointer faces the top thick line marking.



## LOSSES AND EFFICIENCY.

In alternator, the following losses occur

- Copper loss  
occurs only in armature winding and field coils.
- Core loss.  
The losses due to eddy current and hysteresis.
- Friction and windage losses.  
The losses due to the bearings and brush friction.
- Load loss  
The armature leakage flux causes eddy current and hysteresis loss in the iron surrounding the armature conductors.

$$\begin{aligned} \text{Alternator Efficiency} &= \frac{kVA \times \text{p.f.}}{kVA \times \text{p.f.} + \text{Losses}} \\ &= \frac{\text{output}}{\text{output} + \text{Losses}} \end{aligned}$$

For maximum efficiency

$$\text{Iron loss} = \text{Copper loss}$$

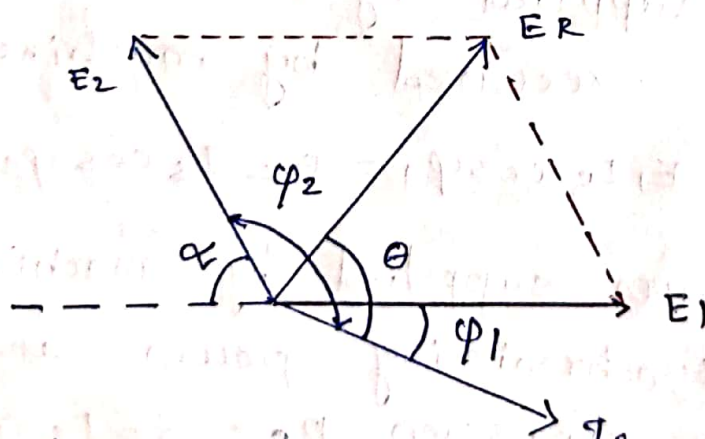
## SYNCHRONIZING CURRENT.

→ When two alternators are in exact synchronism, the two alternators have equal induced emfs and are in exact phase opposition.

→ No circulating current flows round the load circuit.



→ When the induced emfs of the two alternators are equal but not in exact phase opposition, their resultant emf acts round the local circuit and causes flow of current called the synchronizing current.



Resultant emf  $E_R = 2E \cos \left( \frac{[180^\circ - \alpha]}{2} \right)$   
 $= 2E \cos \left( 90^\circ - \frac{\alpha}{2} \right)$



$$= 2E \sin \frac{\alpha}{2}$$

$$= 2E \times \frac{\alpha}{2} = E\alpha$$

$$I_s = \frac{E_R}{Z} = \frac{E\alpha}{Z}$$

$$\theta = \tan^{-1} \frac{X_s}{R}$$

synchronizing current  $I_s = \frac{E\alpha}{X_s}$

$I_s$  lagging behind  $E_R$  by  $90^\circ$ .

Synchronizing power.

Machine 1 supplies power equal to  $E_1 I_s \cos \phi_1$  and machine 2 receives power equal to  $E_2 I_s \cos \phi_2$ .

power supplied by machine 1 = power received by machine 2 + copper loss

$$E_1 I_s \cos \phi_1 = E_2 I_s \cos \phi_2 + \text{copper loss}$$

The power supplied by machine 1 is called the synchronizing power and is given by the expression  $P_s = E_1 I_s \cos \phi_1$

$$= E \times \frac{E\alpha}{X_s} = \frac{\alpha E^2}{X_s}$$

$$\text{Total synchronizing power} = 3P_s = \frac{3\alpha E^2}{X_s}$$

## Synchronizing Torque.

$$3 P_s = T_s \times \frac{2\pi N_s}{60}$$

$$T_s = \frac{3 P_s \times 60}{2\pi N_s}$$

$$\text{Where } N_s = \frac{120f}{P}$$

Example: A 5 MVA, 10,000 V, 1500 rpm, 3φ, 50 Hz 4 pole alternator is operating on an infinite bus bar. Find synchronizing power per mechanical degree of angular displacement under no load excitation given  $X_s = 20\%$ .

Given data

Alternator rating = 5 MVA

terminal voltage = 10,000 V

frequency = 50 Hz

speed = 1500 rpm

poles  $P = 4$

$X_s = 20\%$

solution:

$$\text{Voltage / phase} = \frac{10,000}{\sqrt{3}}$$

$$= 5773.5 \text{ V}$$

$$\text{Full load current } I = \frac{\text{kVA}}{\sqrt{3} \text{ kV}} \times 10^3$$

$$= \frac{\text{Power}}{\sqrt{3} \times V_{ph}}$$



$$= \frac{5 \times 10^6}{53 \times 10,000}$$

$$= 288.67 \text{ A}$$

$$I X_s = 20\% \text{ of } 5773.5 \text{ V}$$

$$\therefore X_s = \frac{0.2 \times 5773.5}{288.67} = 4 \Omega$$

$$\text{Synchronizing power } P_{sy} = \frac{3 \times \alpha E^2}{X_s}$$

$\alpha = 1^\circ \text{ mech}$

$$\alpha (\text{electrical}) = \frac{\alpha \times P}{2} = 1 \times \frac{4}{2} = 2^\circ$$

$$\alpha = 2 \times \frac{\pi}{180^\circ} = \frac{\pi}{90} \text{ elect-rad}$$

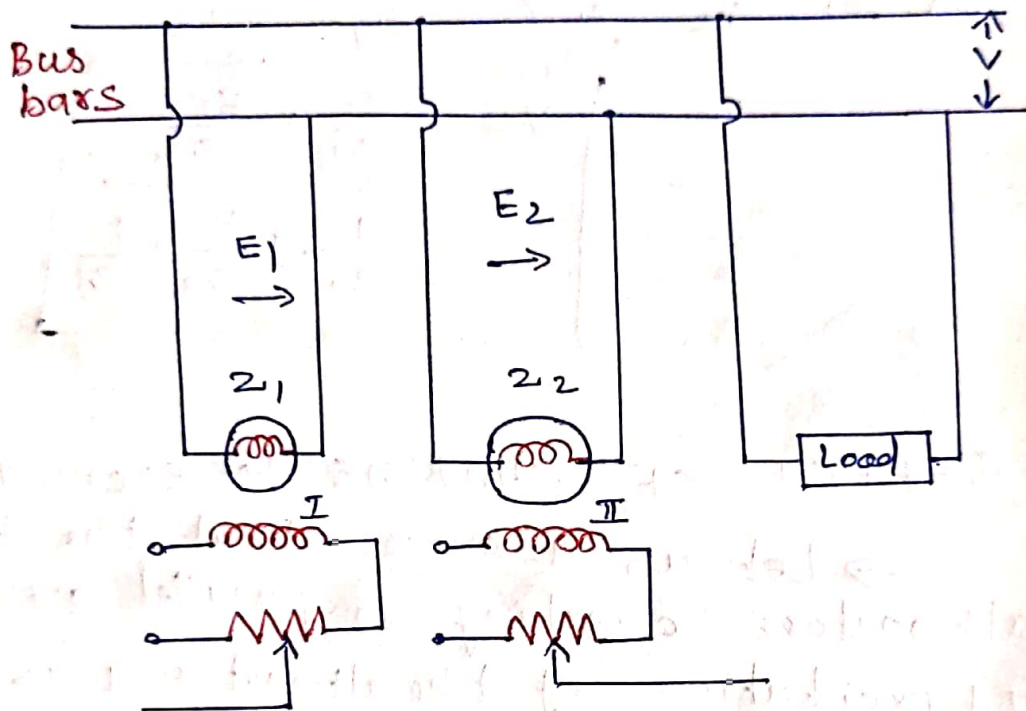
$$P_{sy} = \frac{3\pi \left(\frac{\pi}{90}\right) \times 5773.5^2}{4}$$

$$P_{sy} = 872.66 \text{ kW}$$

Example :-  
(H-w) A 3000 kVA 16 pole alternator runs at 1000 rpm in parallel with other machines on 3300 V bus-bars. Its synchronous reactance is 25%. Calculate the synchronizing power for one mechanical degree of displacement and the corresponding synchronizing torque.

## LOAD SHARING BETWEEN TWO ALTERNATOR.

Consider two machines with identical speed load characteristics running in parallel with a common terminal voltage of  $V$  and load impedance  $Z_L$ .



Let the generated emf of machines I and II be  $E_1$  and  $E_2$  respectively and phase impedances be  $Z_1$  and  $Z_2$  respectively.

Terminal voltage of machine I,  $V = E_1 - I_1 Z_1$

Terminal voltage of machine II,  $V = E_2 - I_2 Z_2$

$$V = Z_L I = (I_1 + I_2) Z_L$$

$$I_1 = \frac{E_1 - V}{Z_1}, \quad I_2 = \frac{E_2 - V}{Z_2}$$



$$I_1 + I_2 = \frac{E_1 - V}{Z_1} + \frac{E_2 - V}{Z_2}$$

$$\frac{V}{Z} = \frac{E_1 - V}{Z_1} + \frac{E_2 - V}{Z_2}$$

$$V \left( \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z} \right) = \frac{E_1}{Z_1} + \frac{E_2}{Z_2}$$

$$V = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z}}$$

### EFFECT OF CHANGE IN EXCITATION.

→ Let us assume that the two alternators operating in parallel are identical. If excitation of the alternator 1 is increased  $E_1 > E_2$ ,  $I_c$  (circulating current) flows through the armature and round the busbars. From the figure  $I_c$  is added vectorially to the load current of alternator 1 and subtracted from the load current of alternator 2, which causes a change in load current.

power factor for alternator 1 →  $\cos \phi_1$

power factor for alternator 2 →  $\cos \phi_2$

$$\cos \phi_1 > \cos \phi_2$$

→ Although the two machines deliver the load current at different power factor

it has no effect on kW output, but kVAR supplied by alternator 1 is increased whereas kVAR supplied by alternator 2 is decreased as shown in figure.

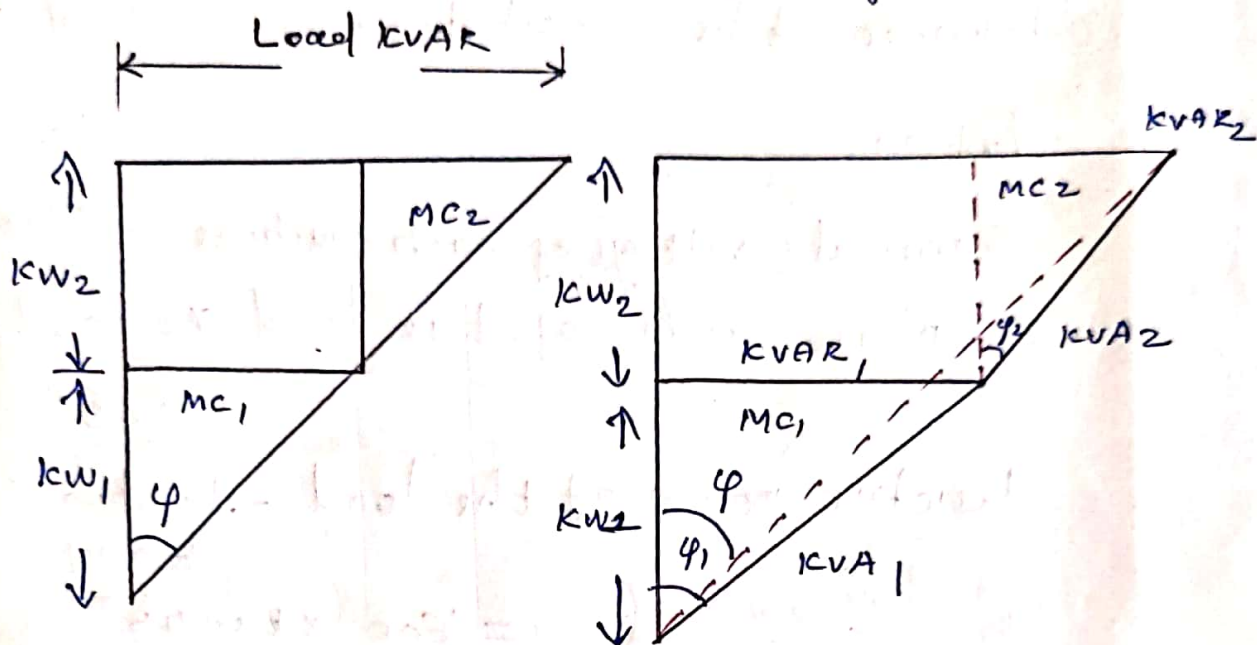


Fig. Effect of change in excitation.

Example :- The following data pertains to two similar alternators. 3 phase, 6600 V, 60 Hz, 200 kW  $\Delta$  connected, Resistance / phase =  $1.05 \Omega$  synchronous reactance per phase =  $5 \Omega$  saturation curve.

Field current in A	150	200	250	300
Terminal voltage in V	5600	6490	7000	7400
	350	500		
	7750	8500		



when operating in parallel with a terminal voltage of 6600 V, the first machine supplies 90 A at 0.8 lag pf. If the load pf is 0.707 lag and total load is 1600 kW, determine the excitation of second alternator.

solution.

Terminal voltage of each machine = 6600 V

phase angle of the load  $\phi = \cos^{-1}(0.707)$   
 $= 45^\circ$

Reactive power of the load = Load in kW  $\times$   
 $\tan \phi$

$$= 1600 \times \tan 45^\circ$$

$$= 1600 \text{ kVAR.}$$

kW supplied by machine 1 =  $\frac{S_1 V \cos \phi}{1000}$

$$= \frac{S_1 \times 6600 \times 90 \times 0.8}{1000}$$

$$= 823 \text{ kW.}$$

Reactive power supplied by machine 1,

$$= \text{load in kW} \times \tan \phi,$$

$$= 823 \times \tan(\cos^{-1} 0.8)$$

$$= 617.2 \text{ kVAR.}$$

power supplied by machine 2,

$$= 1600 - 823 = 777 \text{ kW.}$$

Reactive power supplied by machine 2.

$$= 1600 - 617.2 = 982.8 \text{ kVAR.}$$

Since power supplied =  $\frac{\sqrt{3} V I_2 \cos \phi_2}{1000}$

$$I_2 \cos \phi_2 = \frac{\text{Power supplied in (kW)} \times 1000}{\sqrt{3} V}$$

$$= \frac{777 \times 1000}{\sqrt{3} \times 6600} = 67.97 \text{ A.}$$

since reactive power supplied

$$= \frac{\sqrt{3} V I_2 \sin \phi_2}{1000}$$

$$I_2 \sin \phi_2 = \frac{\text{reactive power in (kVAR)} \times 1000}{\sqrt{3} V}$$

$$= \frac{982.8 \times 1000}{\sqrt{3} \times 6600} = 85.97 \text{ A.}$$

$$\text{Phase voltage} = \frac{6600}{\sqrt{3}} = 3810 \text{ V}$$

$$\begin{aligned} \text{Now } E_0 &= \sqrt{\left( V + I_2 R \cos \phi_2 + I_2 X \sin \phi_2 \right)^2 + \left( I_2 X \cos \phi_2 - I_2 R \sin \phi_2 \right)^2} \\ &= \sqrt{\left( 3810 + 67.97 \times 1.05 + 85.97 \times 5 \right)^2 + \left( 67.97 \times 5 - 85.97 \times 1.05 \right)^2} \\ &= 4318.4 \text{ V.} \end{aligned}$$



$$\text{So } E_{\text{Line}} = \sqrt{3} \times 4318.4 = 7480 \text{ V.}$$

From saturation curve excitation required to induce an emf of 7480 V = 308 A.

## SALIENT POLE MACHINE : TWO REACTION THEORY.

Andrew Blondel had proposed the two reaction theory which resolves the given armature mmfs into two mutually perpendicular components as follows.

→ One component is located along the axis of salient pole rotor, known as direct axis component. (d-axis)

→ The other component is located perpendicular to the axis of salient pole rotor known as quadrature axis component. (q-axis)

Because of non uniformity reluctance, mmf of the armature is divided into two components,

i) a direct component

ii) quadrature component.

When armature current is in phase with the excitation voltage, the entire mmf of the armature acts at right angles to the axis of the salient poles and therefore, all the armature mmf is in quadrature.

When the phase difference between armature current and excitation voltage is in between  $0$  and  $90^\circ$ , the armature will have both a direct



acting and a quadrature components.

Reactance voltages are respectively  $I_d X_{ad}$  and  $I_q X_{aq}$

$I_d, I_q \rightarrow$  armature current along direct and quadrature axis respectively.

direct axis synchronous reactance

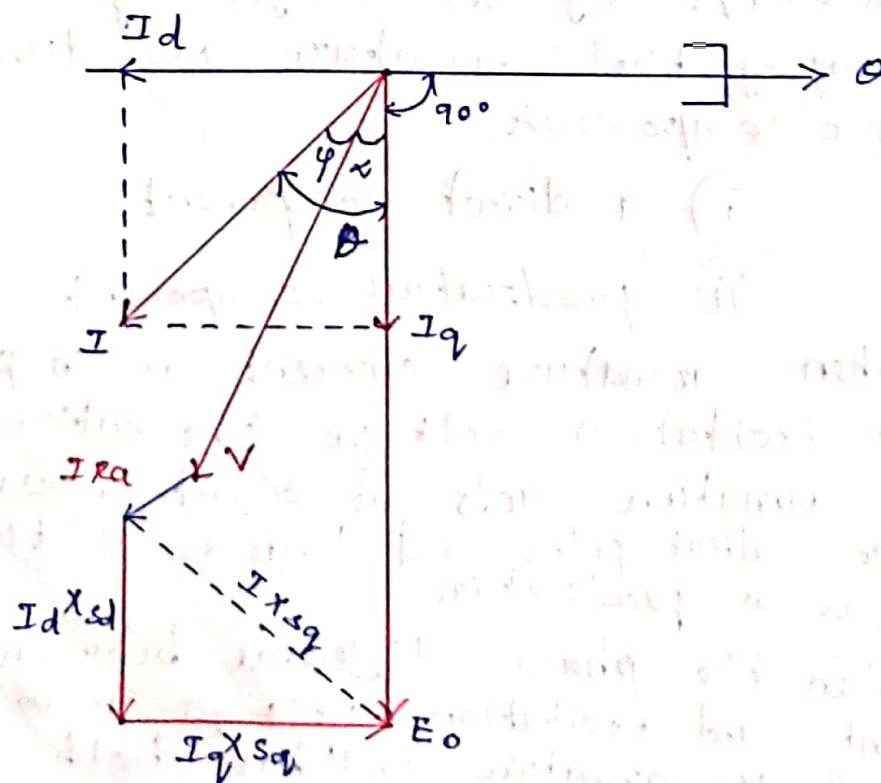
$$X_{sd} = X_{ad} + X_L$$

quadrature axis synchronous reactance

$$X_{sq} = X_{aq} + X_L$$

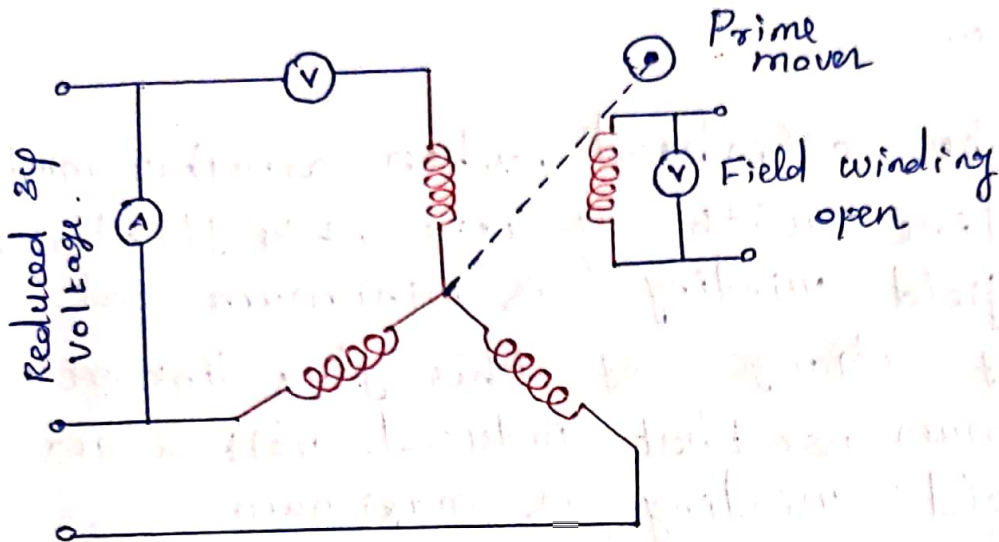
Voltage equation.

$$V = E_0 - I R_a - I_d X_{sd} - I_q X_{sq}$$



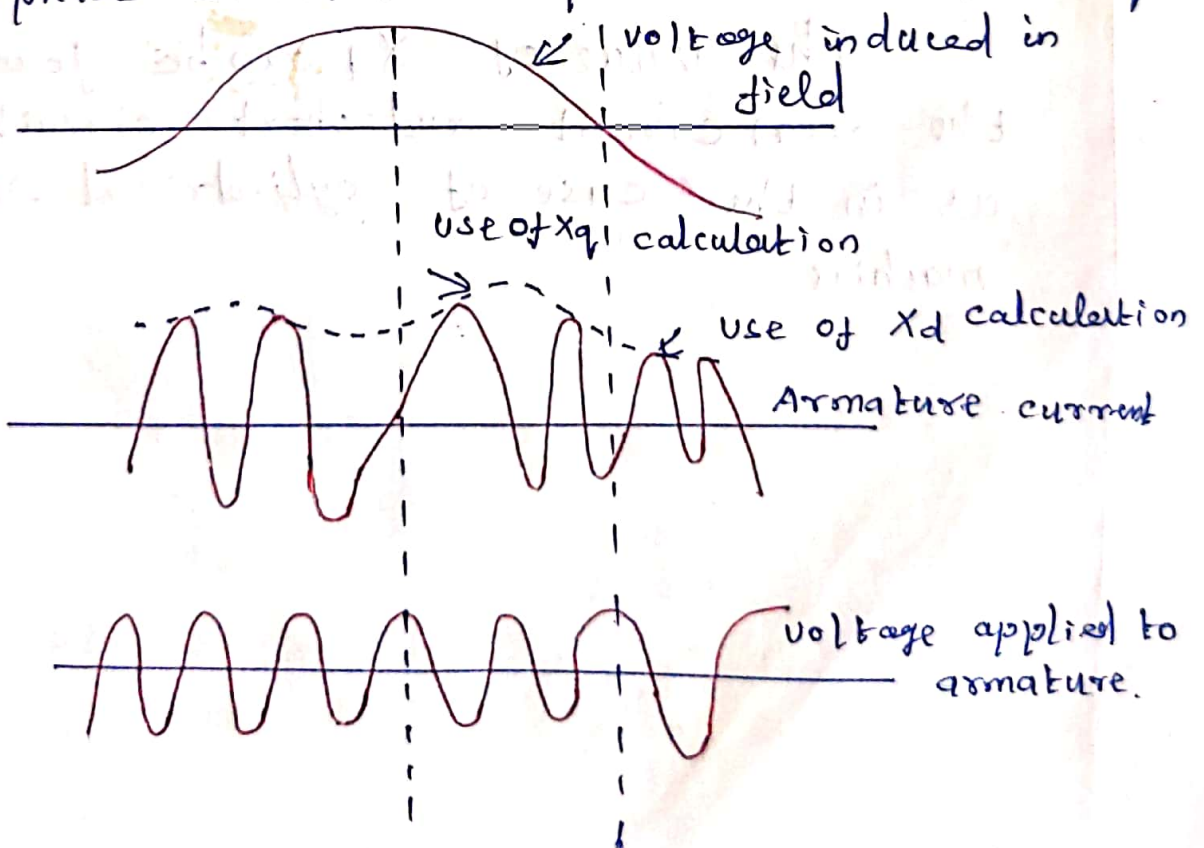
# MEASUREMENT OF $X_d$ AND $X_q$ : SLIP TEST.

The direct axis and quadrature axis reactance can be measured by slip test.



The ratio of armature terminal voltage per phase to armature phase current gives  $X_d$ .

The ratio of armature terminal voltage per phase to armature phase current gives  $X_q$ .





When the armature mmf is in line with field poles, the armature flux linkage with field winding is maximum and rate of change of this flux linkage is zero.

On the other hand, when armature mmf is in line with  $q$ -axis, the flux linkage with field winding is minimum and rate of change of this flux linkage is maximum, so that induced voltage across the field winding is maximum.

The slip should be very small, so that inertia of moving parts of instruments does not cause errors in measurement.

The value of  $X_d$  can be found from the open circuit and short circuit tests, as in the case of cylindrical motor machine.



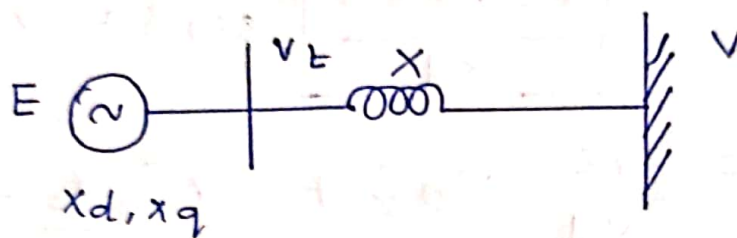
## POWER ANGLE CHARACTERISTICS OF SALIENT POLE GENERATOR.

Fig shows a salient pole synchronous generator feeding an infinite bus,

$V$  - bus voltage

$E$  - excitation voltage of generator.

$V_E$  - terminal voltage of generator.



Total d and q-axis reactances are,

$$X_d = X + X_d$$

$$X_q = X + X_q$$

The total power delivered by the generator can be found by resolving the bus voltage  $V$  into  $V \sin \delta$  and  $V \cos \delta$ .

$$\text{Thus } P = 3 I_d V \sin \delta + 3 I_q V \cos \delta. \quad \text{--- (1)}$$

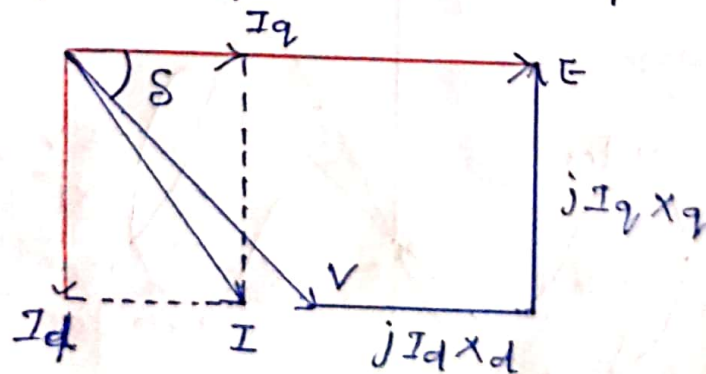


Fig. phasor diagram of salient pole alternator feeding an infinite bus.



The current  $I_d$  and  $I_q$  are given by,

$$I_d = \frac{E - V \cos \delta}{X_d} \quad \text{--- (2)}$$

$$I_q = \frac{V \sin \delta}{X_q} \quad \text{--- (3)}$$

substituting equations 2 and 3 in eqn 1.

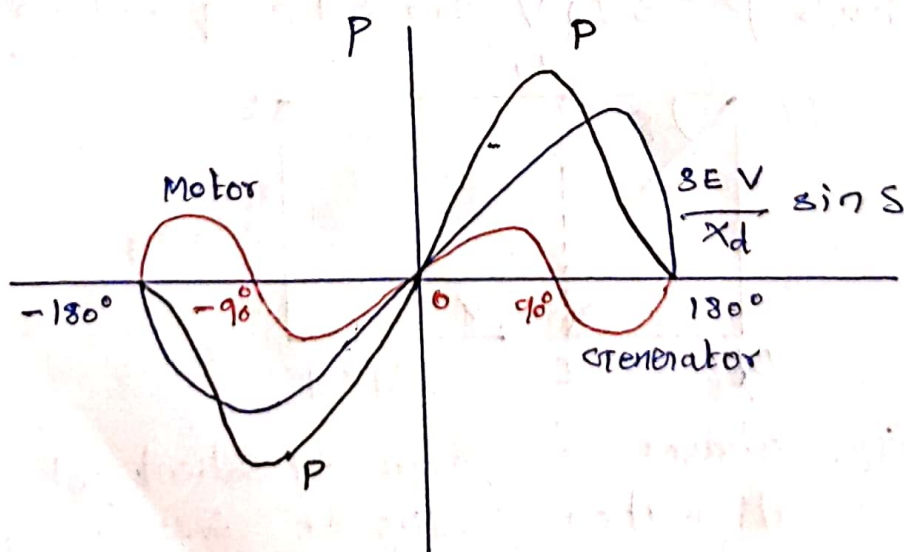
$$P = 3V \left[ \frac{E - V \cos \delta}{X_d} \right] \sin \delta + 3V \left[ \frac{V \sin \delta}{X_q} \right] \cos \delta$$

$$P = \frac{3VE}{X_d} \sin \delta + 3V^2 \left( \frac{X_d - X_q}{2X_d X_q} \right) \sin 2\delta \quad \text{--- (4)}$$

Since torque developed equals power output divided by  $\omega_s$ , torque is given by,

$$\text{Torque} = \frac{3VE}{\omega_s X_d} \sin \delta + \frac{3V^2 (X_d - X_q)}{2\omega_s X_d X_q} \sin 2\delta$$

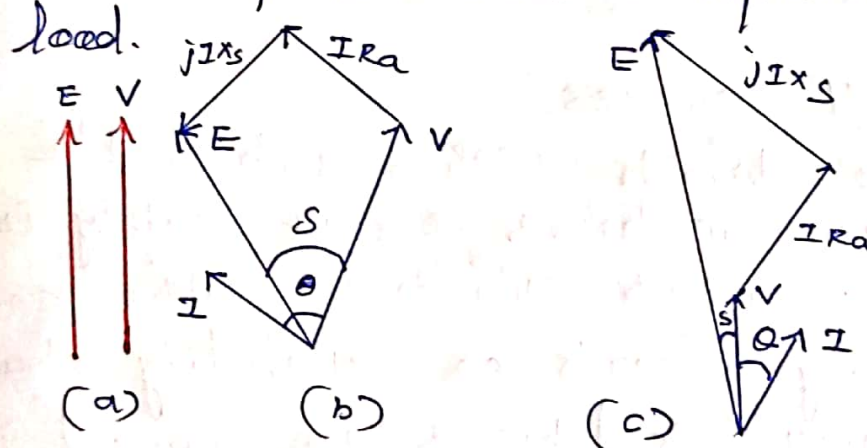
--- (5)



# OPERATION OF ALTERNATOR ON INFINITE BUS BARS.

Most of the alternators feed power into systems which are composed of a large number of interconnected alternators. The size of the system is so large that the addition or removal of one alternator does not affect the system voltage and frequency. Such a large system, operating at constant voltage and frequency, is known as an infinite bus bar system.

An alternator working on infinite bus bars will become a motor if its excitation is maintained but the prime mover is replaced by a mechanical load.



The power angle  $\delta$  plays an important role in the operation of the machine. When operation alters from generator to motor,  $\delta$  changes from positive to negative.

After synchronizing, an alternator is loaded by adjusting the governor setting to admit more steam. This tends to advance the rotor and load current flows from the machine to



The bus bars maintaining an exact balance between the mechanical input and electrical output.

The following conditions when an alternator is operating in parallel with an infinite bus system,

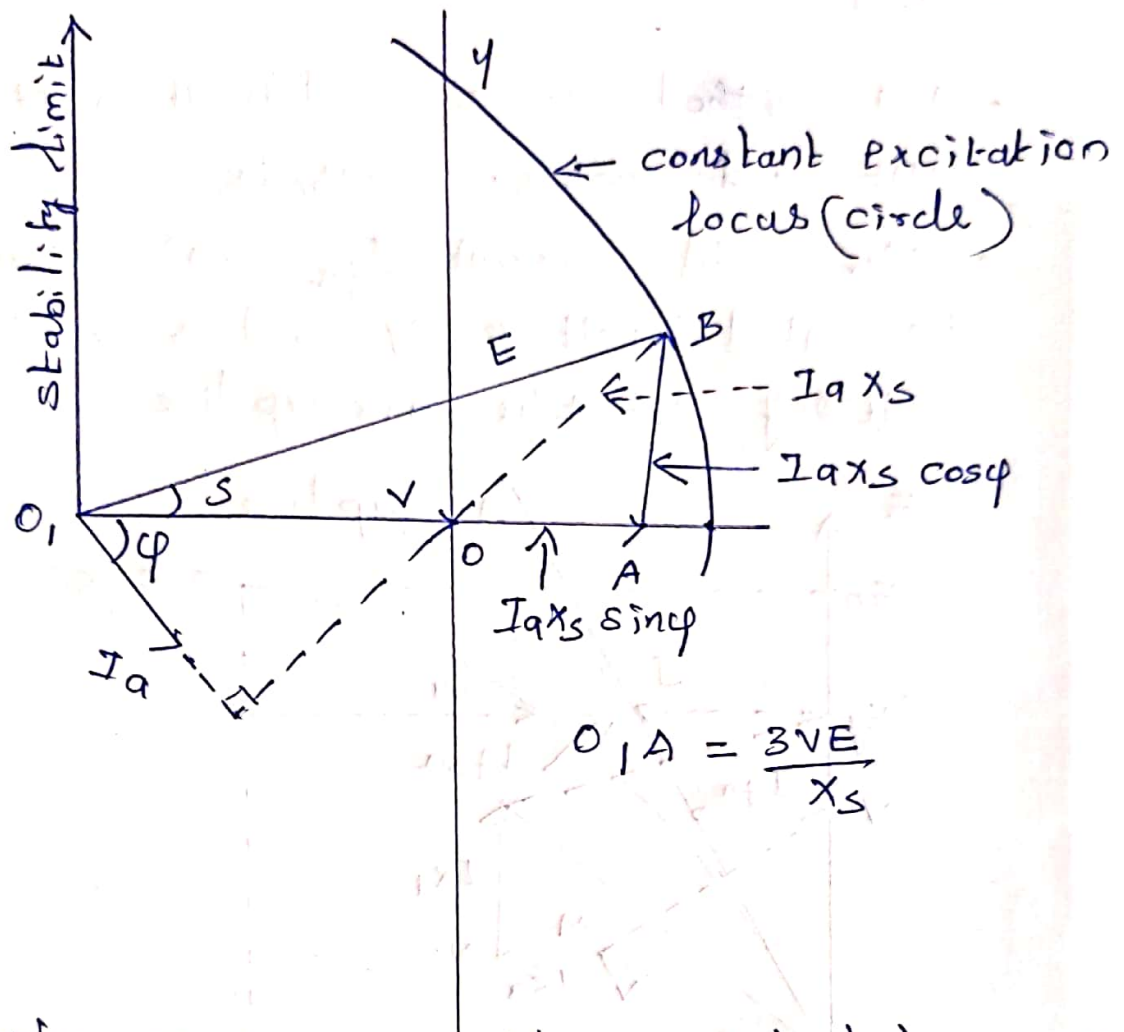
- i) The terminal voltage and frequency remain constant at the system voltage and frequency.
- ii) The governor set points of alternator control the real power output of the alternator.
- iii) The field current of the alternator controls its reactive power output and power factor.

#### CAPABILITY CURVES.

The limits within which the synchronous machines operate safely is called capability curves which are also known as operating charts or capability charts.

The following conditions are required.

- The MVA loading must be less than the rating of the alternator.
- The MW loading must be less than the rating of the prime mover.
- The field current must not cross the value determined by the field heating.
- The angle  $\delta$  must be less than  $90^\circ$  for steady state stability operation.



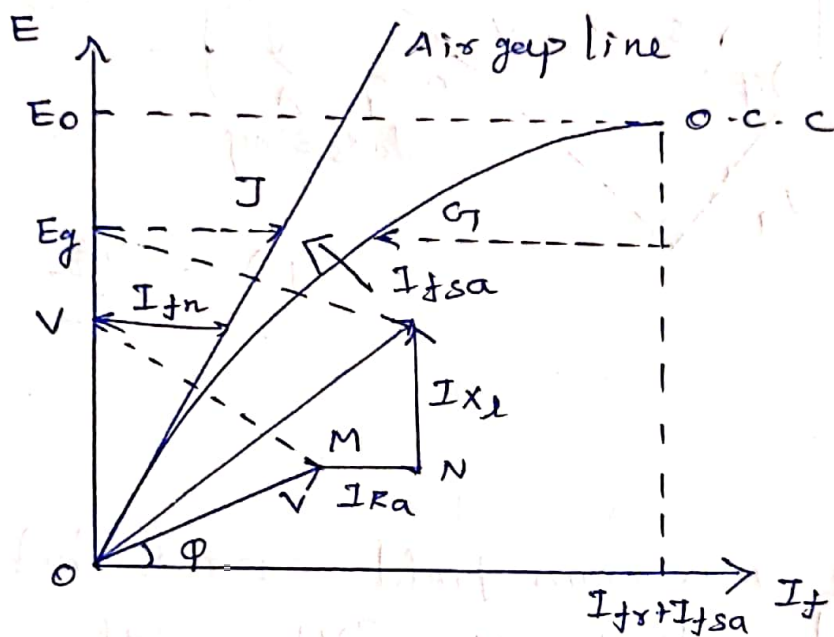
The maximum field current between  $P$  and  $q$  and radius is  $\frac{3VE}{x_s}$  with centre  $O_1$ .

The operation of the alternator is limited by the power factor of the prime mover between  $r$  and  $s$ . The operation of alternator is limited by the practical stability limit between  $s$  and  $t$ . The practical stability limit is usually taken as 10% less than the theoretical stability limit for a safety margin.



# AMERICAN STANDARDS ASSOCIATION (ASA) METHOD.

- ASA method is a modification of the MMF and potier methods.
- The field current  $I_{fn} (=OA)$  corresponds to rated voltage  $V$  and is found out by referring to the air gap line.



By referring to open circuit characteristics and air gap line the difference in field current  $J$  or is found out corresponding to the voltage  $E_g$ .

$$BC = J \sigma = (I_{f_{sa}})$$

$$\% \text{ Regulation} = \frac{E_0 - V}{V} \times 100$$

ASA method gives more accurate results and is applicable for both salient and non salient pole synchronous machine.

## UNIT II

### SYNCHRONOUS MOTOR.

- Principle of operation
- Torque equation
- operation on infinite bus bar
- V and inverted V curves
- Power input and power developed equations
- starting methods
- current loci for constant power input, constant excitation and constant power developed.
- Hunting
- Natural frequency of oscillation.
- damper windings
- synchronous condenser.



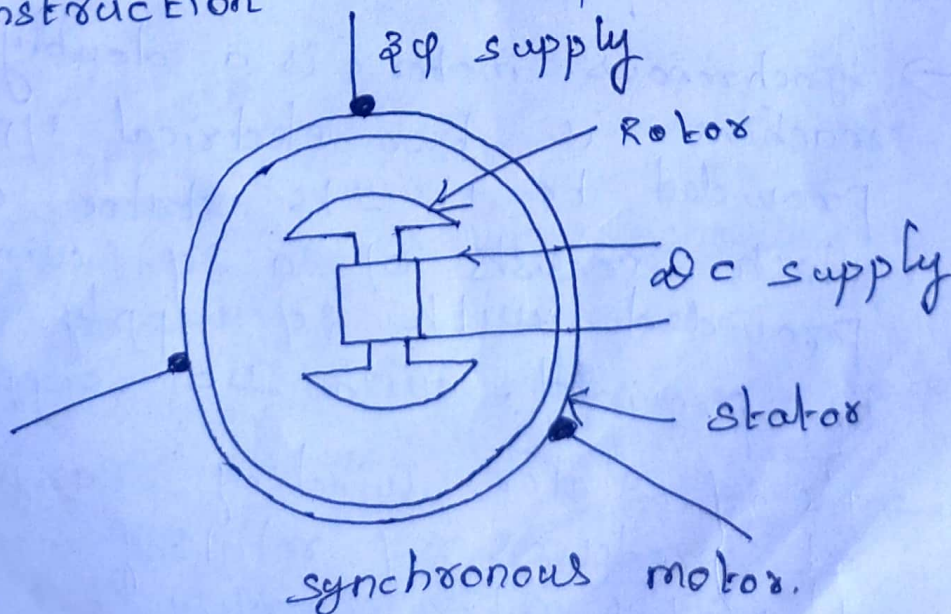
④  
SYNCHRONOUS MOTOR.

A synchronous electric motor is an AC motor in which, at steady state, the rotation of the shaft is synchronized with the frequency of the supply current; the rotation period is exactly equal to an integral number of AC cycles.

Synchronous motor is called so because the speed of the rotor of this motor is same as the rotating magnetic field. It is basically a fixed speed motor because it has only one speed, which is synchronous speed and therefore no intermediate speed is there or in other words it is in synchronism with the supply frequency.

$$N_s = \frac{120f}{P}$$

Construction





Normally its construction is a motor similar to that of a  $\phi$  IM, except the fact that the rotor is given dc supply. The stator is given  $\phi$  supply. The reason of which have some features

\* Synchronous motors are inherently not self starting. They require some external means to bring their speed close to synchronous speed to before they are synchronized.

\* The speed of operation of is in synchronism with the supply frequency and hence for constant supply frequency they behave as constant speed motor irrespective of load condition.

\* This motor has the unique characteristic of operating under any electrical power factor. This makes it being used in electrical power factor improvement.

Principle & operation.

→ synchronous motor is a doubly excited machine i.e. two electrical i/p's are provided to it. Its stator winding which consists of a  $\phi$  winding is provided with  $\phi$  supply and rotor is provided with dc supply.

→ The  $\phi$  stator winding carrying  $\phi$  c.t. produces  $\phi$  rotating magnetic flux. The rotor carrying dc supply



(2)

also produces a constant flux. Considering the frequency to be 50 Hz, from the above relation we can see that the  $\phi$  rotating flux rotates about 2000 revolution in 1 min or 50 revolution in 1 sec.

→ At a particular instant rotor and stator poles might be of same polarity (N-N or S-S) causing repulsive force on rotor and the very next second it will be N-S causing attractive force. But due to inertia of the rotor, it is unable to rotate in any direction due to attractive or repulsive force and remain in standstill condition. Hence it is not self starting.

→ To overcome this inertia, rotor is initially fed some mechanical i/p which rotates it in same direction as magnetic field to a speed very close to synchronous speed. After some time magnetic locking occurs and the synchronous motor rotates in synchronism with the frequency.

Methods of starting of synchronous motor

1. Synchronous motors are mechanically coupled with another motor. It could be either  $\phi$  I.M or DC shunt motor.



DC excitation is not fed initially. It is rotated at speed very close to its synchronous speed and after that DC excitation is given. After some time when magnetic locking takes place supply to the external motor is cut off.

→ Damper winding: In case, synchronous motor is of salient pole type, additional winding is placed in rotor pole face. Initially when rotor is standstill relative speed b/w damper winding and rotating air gap flux is large and an emf is induced in it which produces the required starting torque. As speed approaches synchronous speed, emf and torque is reduced and finally when magnetic locking takes place, torque also reduces to zero. Hence in this case synchronous is first run as  $\approx \phi$  I.M using additional winding and finally it is synchronized with the frequency.

→ Applications:

1. Synchronous motor having no load connected to its shaft is used for power factor improvement. Owing to its characteristic to behave at any electrical power factor, it is used in power system in situations where static capacitors are expensive.
2. Synchronous motor finds application where operating speed is less (around 500 rpm) and high power is required.



(8)

→ For power requirement from 35 kW to 2500 kW, the size, weight and cost of the corresponding 3φ IM is very high.  
- Ex: reciprocating pumps, compressor, rolling mills etc.

Magnetic locking.

Force of attraction exists b/w two magnets in such case when one of the magnet is moved in one direction, the other magnet also moves in the same direction with the same speed with which the first magnet was moved. The above principle is called magnetic locking.

Ex: when pure inductive load is connected to the alternator, what is the effect of armature reaction?  
demagnetization.



# TORQUE EQUATION OF THE SYNCHRONOUS MOTOR.

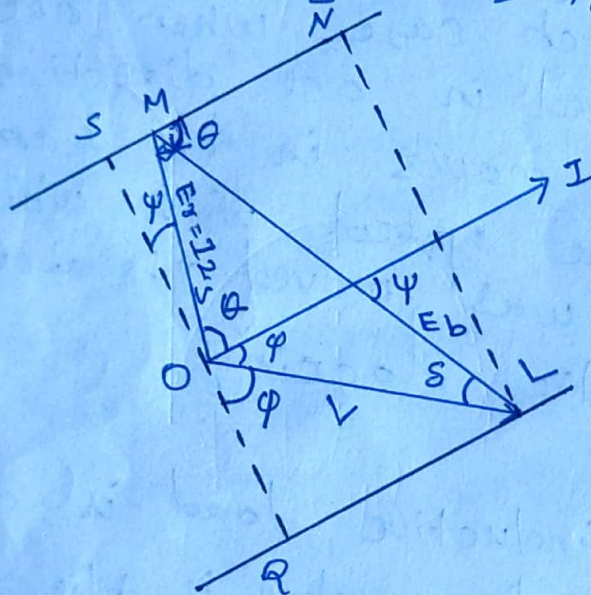
$OL =$  supply voltage / phase

$I =$  Armature current

$LM =$  Back emf at a load angle of  $\delta$ .

$OM =$  Resultant voltage,  $E_r$ .

$$E_r = I Z_s \text{ (or } I X_s \text{ if } R_a \text{ is negligible)}$$



- $I$  lags / leads  $V$  by an angle  $\phi$  and lags behind  $E_r$  by an angle  $\theta$ .

$$\therefore \theta = \tan^{-1} \left( \frac{X_s}{R_a} \right)$$

- Line  $NS$  is drawn at angle  $\theta$  to  $LM$ .
- $LN$  and  $QS$  are perpendicular to  $NS$ .
- Mechanical power developed per phase in the motor.

$$P_{mech} = E_b I \cos \psi$$

$$\Delta OMS, MS = I Z_s \cos \phi$$



$$M_s = N_s - N_m = LQ - NM$$

$$I Z_s \cos \psi = V \cos(\theta - \delta) - E_b \cos \alpha$$

$$\text{(or)} \quad I \cos \psi = \frac{V}{Z_s} \cos(\theta - \delta) - \frac{E_b}{Z_s} \cos \alpha$$

$$\bullet \quad P_{\text{mech}} / \text{ph} = E_b \left[ \frac{V}{Z_s} \cos(\theta - \delta) - \frac{E_b}{Z_s} \cos \alpha \right]$$

$$P_{\text{mech}} / \text{ph} = \frac{E_b V}{Z_s} \cos(\theta - \delta) - \frac{E_b^2}{Z_s} \cos \alpha$$

This is the expression for the mechanical power developed in terms of load angle ( $\alpha$ ) and the internal angle  $\delta$  of the motor for a constant voltage  $V$  and  $E_b$ .

Maximum power developed:

Condition for maximum power developed can be found by differentiating the above expression with respect to load angle and then equating it to zero.

$$\frac{dP_{\text{mech}}}{d\alpha} = -\frac{E_b V}{Z_s} \sin(\theta - \delta) = 0$$

$$\therefore \sin(\theta - \delta) = 0 \quad \text{(or)} \quad \theta = \delta$$

$\therefore$  value of maximum power

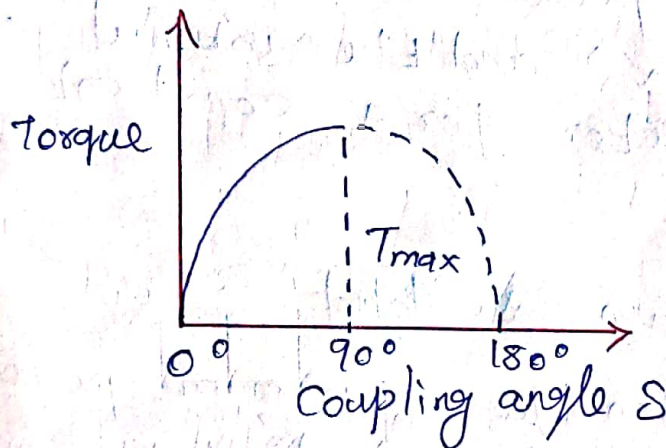
$$\begin{aligned} (P_{\text{mech}})_{\text{max}} &= \frac{E_b V}{Z_s} - \frac{E_b^2}{Z_s} \cos \delta \\ &= \frac{E_b V}{Z_s} - \frac{E_b^2}{Z_s} \cos \alpha \end{aligned}$$

(4)

→ This shows that the maximum power and hence torque depends on  $V$  and  $E_b$  that is excitation.

→  $\theta$  gets maximum when  $\alpha = 90^\circ$   
Maximum torque will be proportional to the maximum power developed.

→ If  $R_a$  is neglected, then  $Z_s = X_s$   
and  $\theta = 90^\circ$



$$\cos \theta = 0$$

$$P_{\text{mech}} = \frac{E_b V}{X_s} \cos(90^\circ - \delta)$$

$$P_{\text{mech}} = \frac{E_b V}{X_s} \sin \delta$$

$$\delta = 90^\circ$$

$$P_{\text{mech}} = \frac{E_b V}{X_s}$$

→ To determine the value of excitation or induced emf  $E_b$  to give maximum power developed possible, differentiate



with respect to  $E_b$  and equate to zero.

$$\frac{d P_{\text{mech}} (\text{max})}{d E_b} = \frac{V}{Z_s} - \frac{2 E_b \cos \theta}{Z_s} = 0$$

$$E_b = \frac{V}{2 \cos \theta}$$

$\Rightarrow$  substituting  $E_b = \frac{V}{2 \cos \theta}$  in  $(P_{\text{mech}})_{\text{max}}$

$$(P_{\text{mech}})_{\text{max}} = \frac{V^2}{2 Z_s \cos \theta} - \frac{V^2}{4 Z_s \cos \theta}$$

$$= \frac{V^2}{4 Z_s \cos \theta} = \frac{V^2}{4 R_a}$$

where  $R_a$  = Effective resistance of the motor.

$$\text{Hence } (P_{\text{mech}})_{\text{max}} = \frac{V^2}{4 R_a}$$

Power flow in a synchronous motor.

Let

$R_a$  = Armature resistance / phase

$X_s$  = Synchronous reactance / phase

$Z_s$  = Synchronous impedance

$$Z_s = R_a + j X_s$$

$$I_a = \text{Armature current} = \frac{E_r}{Z_s}$$

$$= \frac{V - E_b}{Z_s}$$

$$\theta = \text{Internal angle} = \tan^{-1} \left( \frac{X_s}{R_a} \right)$$

If  $R_a$  is negligible, then  $\theta = 90^\circ$

Input power to the motor  $P_{ph} = VI \cos \phi$

Total input power for a star connected 3 phase synchronous motor,

$$= \sqrt{3} V_L I_L \cos \phi$$

The mechanical power developed in the rotor,

$$P_m = E_b I_a \cos(\delta - \phi) \text{ per phase.}$$

Power wasted on the armature or armature copper loss per phase  $= I_a^2 \cdot R_a$

$$\text{Input power per phase } P_{in} = P_m + I_a^2 \cdot R_a$$

$$\text{Mechanical power in the rotor } P_m = P_{in} - I_a^2 \cdot R_a$$

$$\text{For three phases } P_m = \sqrt{3} V_L I_L \cos \phi - 3 I_a^2 R_a$$

$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

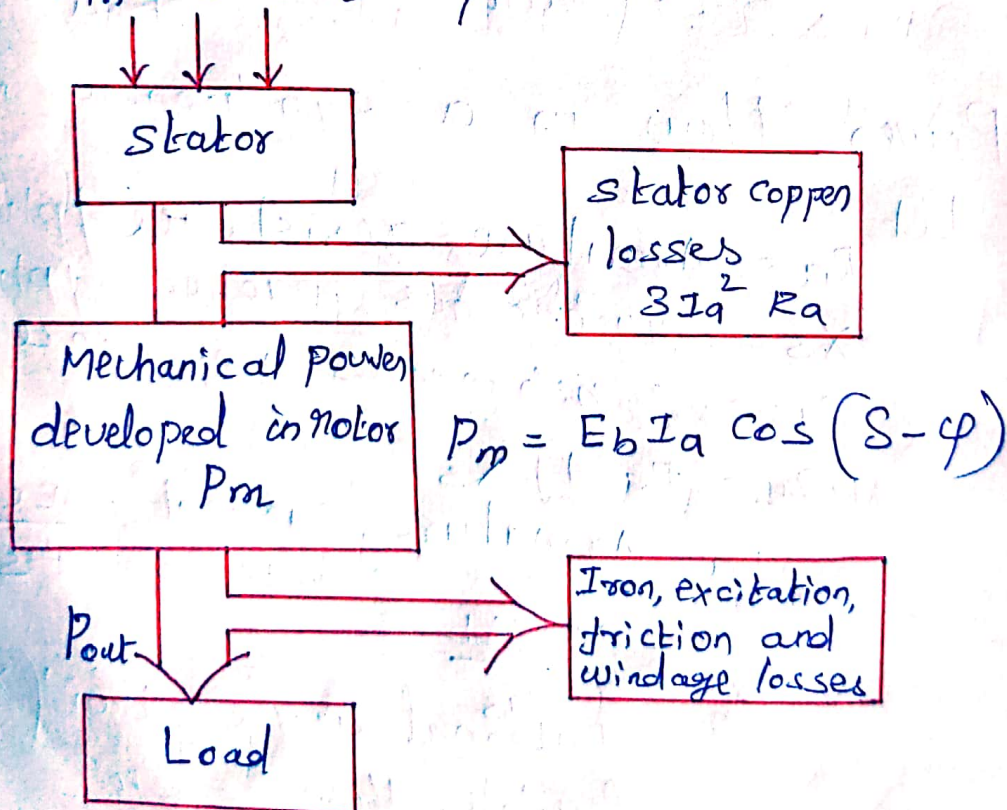


Fig. Power flow diagram of synchronous motor.



Example:- A 6600 V, 3 $\phi$ , star connected synchronous motor draws a full load current of 80 A at 0.8 power factor leading. The armature resistance is 2.2  $\Omega$  and reactance 22  $\Omega$  per phase. If the stray losses of the machine are 3200 W find (i) E.m.f induced (ii) o/p power (iii) Efficiency of the machine.

Solution:

$$\text{phase voltage } V = \frac{6600}{\sqrt{3}} = 3810 \text{ V}$$

Full load current  $I = 80 \text{ A}$  at 0.8 P.F leading

$$R_a \text{ / phase } R_a = 2.2 \Omega$$

$$X_s \text{ / phase } X_s = 22 \Omega$$

Soln:- stray loss = 3200 W

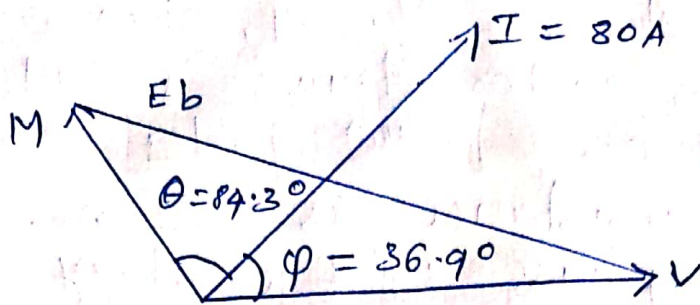
i) e.m.f induced,  $E_b$

$$\cos \phi = 0.8, \phi = 36.9^\circ$$

$$\begin{aligned} Z_s \text{ / phase} &= \sqrt{R_a^2 + X_s^2} \\ &= \sqrt{(2.2)^2 + (22)^2} \\ &= 22.11 \Omega \end{aligned}$$

$$\tan \theta = \frac{X_s}{R_a} = \frac{22}{2.2} = 10$$

$$\theta = \tan^{-1}(10) = 84.3^\circ$$



Impedance drop / Ph,  $E_R = I Z_s = 80 \times 22.11$   
 $= 1768.8 \text{ V}$

Induced emf / Ph,  $E_b = \sqrt{V^2 + E_R^2 - 2VE_R \cos(\theta + \phi)}$   
 $= \sqrt{3810^2 + (1768.8)^2 - 2 \times 3810 \times 1768.8 \cos(84.3^\circ + 36.9^\circ)}$   
 $= 4962.5 \text{ V}$

$\therefore$  Line induced emf  $= \sqrt{3} \times 4962.5$   
 $= 8595 \text{ V}$

(ii) Power o IP

total i/p power  $= \sqrt{3} V_L I_L \cos \phi$

$= \sqrt{3} \times 6600 \times 80 \times 0.8$

$= 731618 \text{ W}$

Total copper loss  $= 3I^2 R_a = 3 \times (80)^2 \times 2.2$

$= 42240 \text{ W}$

Total stray losses  $= 3200 \text{ W}$



$$\begin{aligned} \text{Power O/P} &= \text{I/P Power} - \text{Copper loss} - \text{Stray losses} \\ &= 731618 - 42240 - 3200 \\ P_{\text{out}} &= 686178 \text{ W} \end{aligned}$$

iii) Efficiency,

$$\begin{aligned} \eta &= \frac{\text{O/P Power}}{\text{I/P Power}} \\ &= \frac{686178}{731618} \times 100 \end{aligned}$$

$$\eta = 93.79\%$$

Torque of synchronous motor.

The various torques associated with a synchronous motor are described below.

1. Starting torque :-

It indicates the ability of the motor to accelerate the load. It is also sometimes called break away torque.

- The synchronous motor possesses no self starting torque; yet in modern synchronous motors, by making proper changes in the design of damper windings, almost any reasonable torque can be developed.

## 2. Running Torque:

- Running torque is the torque developed by the motor under running condition.
- It is determined by the output power and speed of the driven machine.
- Peak output power determines the maximum torque that would be required by the driven machine.
- The breakdown or maximum running torque of a motor must be greater than this value in order to avoid stalling of the machine.

## 3. Pull in Torque:

- It pertains to the ability of the machine to pull in to synchronism when changing from induction to synchronous motor operation.



4. Pull out torque:

• It is the maximum torque that the synchronous motor will develop without pulling out of synchronism.

## STARTING METHODS OF SYNCHRONOUS MOTORS.

Various methods of starting of synchronous motors are described below.

1. From DC source

→ When dc supply and dc compound motor are available, the synchronous motor is coupled and started by means of a dc compound motor.

→ The speed of dc motor is adjusted by the speed regulator.

→ The synchronous motor is then excited and synchronized with AC supply mains.

→ At the moment of synchronising, the synchronous motor is switched on with the AC mains and either the dc motor is disconnected from the dc supply mains.



⇒ Now the synchronous machine is operating as a motor, from AC supply mains and dc machine acts as load on it.

⇒ By means of AC Motor:-

→ A small direct coupled induction motor called the pony motor, may be used for starting the synchronous motor unless the motor is required to start against full load torque.

→ Before switching on the AC supply to the synchronous motor, it must be synchronized with the bus-bars.

→ After normal operation is established, the pony motor is sometimes uncoupled from the synchronous motor.

→ Modern machines are usually of the self starting type and are arranged to start as induction motors.

By means of Dampers or aids in the pole faces.

→ The synchronous motor is made self starting by providing a special winding on the rotor poles, known as damper winding or squirrel cage winding.



A synchronous motor having 40% reactance and a negligible resistance is to be operated at rated load at UPF, 0.8 power factor lag and 0.8 power factor lead. What are the values of induced e.m.f?

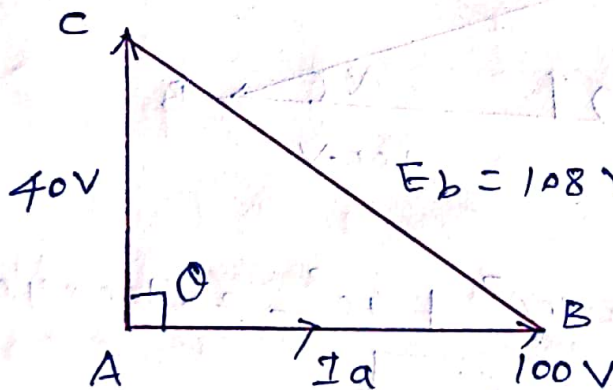
Solution

Let  $V = 100\text{V}$ , Impedance drop  $= I_a X_s = 40\text{V}$

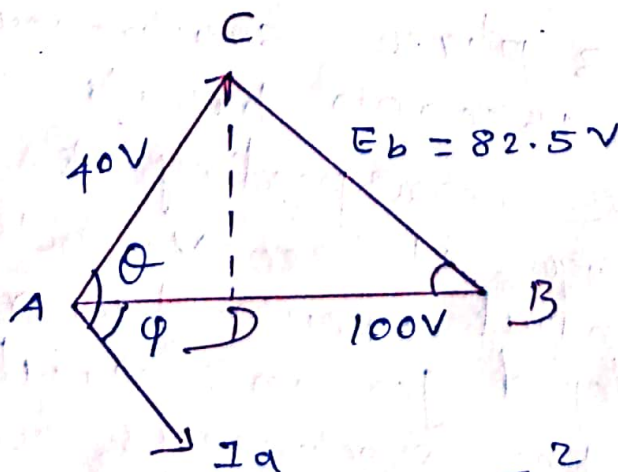
i) UPF

Here  $\theta = 90^\circ$

$$E_b = \sqrt{100^2 + 40^2} = 108\text{V}$$



(ii) 0.8 p.f lagging.



Here,  $\angle CAB = \theta - \phi$   
 $= 90^\circ - 36.86^\circ$   
 $= 53.13^\circ$

$$E_b^2 = 100^2 + 40^2 - 2 \times 100 \times 40 \times \cos 53.13$$

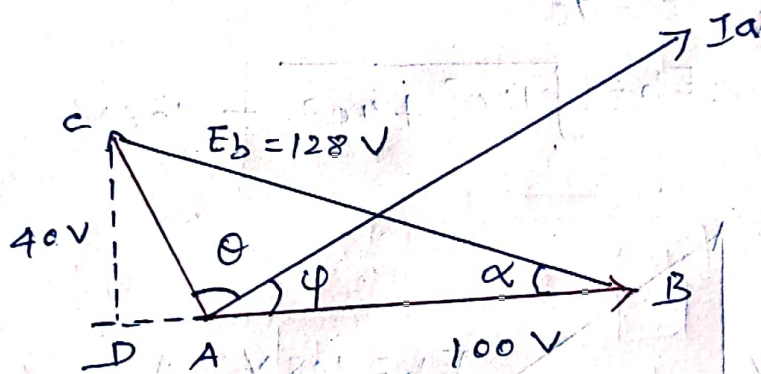
$$E_b = 82.5\text{V}$$

(iii) 0.8 p.f leading

$$\text{Here } \theta + \phi = 90 + 36.86 = 126.86^\circ$$

$$E_b^2 = 100^2 + 40^2 - 2 \times 100 \times 40 \times \cos 126.9$$

$$E_b = 128 \text{ V}$$



$$E_b^2 = 100^2 + 40^2 - 2 \times 100 \times 40 \times \cos 126.9^\circ$$

$$E_b = 128 \text{ V}$$

Example: A 3000 V, 3 phase synchronous motor running at 1500 rpm has its excitation kept constant corresponding to no load terminal voltage of 3000 V. Determine the power input, power factor and torque developed for an armature current of 250 A if the synchronous reactance is  $5 \Omega$  per phase and armature resistance is neglected.



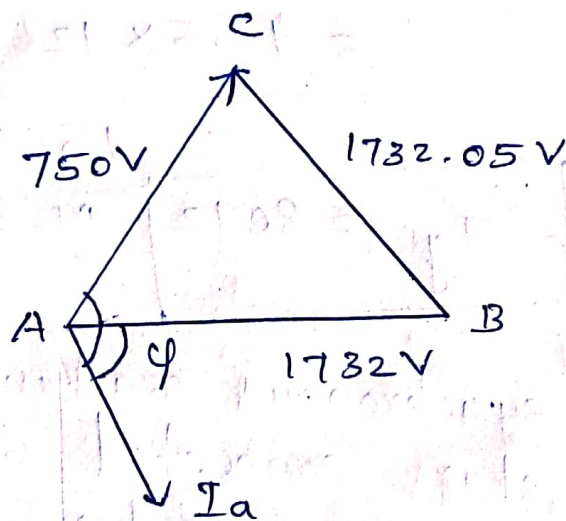
Solution:-

$$\text{Voltage } V_{ph} = \frac{3000}{\sqrt{3}}$$

$$= 1732 \text{ V}$$

$$\text{Induced emf} = 1732 \text{ V}$$

$$\text{Impedance drop} = I_a \times S = 250 \times 5 = 750 \text{ V}$$



From figure, the armature current  $I_a$  is assumed to lag by an angle  $\phi$ . Since  $R_a$  is negligible,  $\theta = 90^\circ$ .

$$\angle CAB = 90 - \phi$$

Considering  $\Delta CAB$ , we have

$$1732.05^2 = 1732^2 + 750^2 - 2 \times 1732 \times 750 \times \cos(90 - \phi)$$

$$\sin \phi = 0.2165$$

$$\phi = 12.5^\circ$$

$$\cos \phi = \cos 12.5 = 0.976 \text{ lag}$$

$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

I/p power  $P_{in} = \sqrt{3} \times 3000 \times 250 \times 0.976$

$$P_{in} = 1267.86 \text{ kW}$$

$$\text{Speed } N_s = 1500 \text{ rpm}$$

$$\text{Torque developed } T_g = \frac{9.55 \times P_{in}}{N_s}$$

$$= \frac{9.55 \times 1267.86 \times 10^3}{1500}$$

$$T_g = 8072 \text{ N-m}$$

Example:- The synchronous reactance per phase of a 3 $\phi$  star connected 6600 V synchronous motor is  $20 \Omega$ . For a certain load, the input is 915 kW at normal voltage and the induced line e.m.f is 8942 V. neglecting resistance determine (i) Line current (ii) Power factor.

Soln:-

Synchronous reactance /ph  $X_s = 20 \Omega$

I/p to motor = 915 kW

supply phase voltage  $V = \frac{6600}{\sqrt{3}} = 3810 \text{ V}$

Induced e.m.f /phase  $E_b = \frac{8942}{\sqrt{3}} = 5163 \text{ V}$

resistance  $R_a = 0$



since induced emf is greater than the supply voltage, the motor must be operating with a leading power factor.

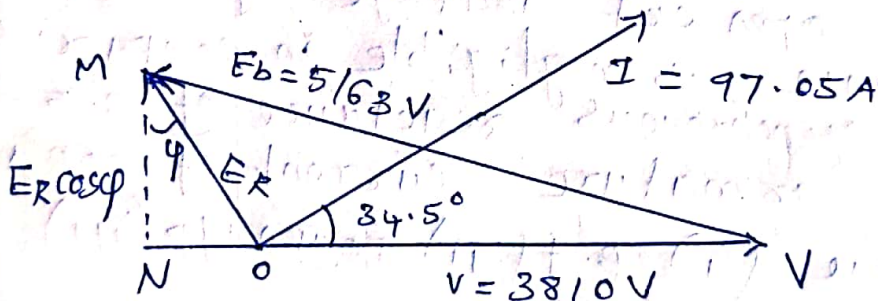
$$\text{power input} = \sqrt{3} V_L I_L \cos \phi$$

$$I_L = \frac{915 \times 1000}{\sqrt{3} \times 6600} = 80 \text{ A}$$

$$\theta = \tan^{-1} \frac{X_s}{R_a} = \tan^{-1} \frac{20}{0}$$

$$= \tan^{-1}(\infty) = 90^\circ$$

$$\begin{aligned} \text{Impedance drop } E_R &= I Z_s = I \sqrt{R_a^2 + X_s^2} \\ &= I \sqrt{0^2 + 20^2} = 20 I \end{aligned}$$



From right-angle,  $\Delta LMN$  we have

$$LM^2 = LN^2 + MN^2$$

$$(5163)^2 = LN^2 + (E_R \cos \phi)^2 = LN^2 + (20 I \cos \phi)^2$$

$$= LN^2 + (20 \times 80)^2 \quad (\because I \cos \phi = 80 \text{ A})$$

$$LN^2 = 5163^2 - 1600^2$$

$$LN = 4909 \text{ V}$$

$$ON = LN - OL = 4909 - 3810 = 1099 \text{ V}$$

$$E_R = OM = \sqrt{ON^2 + MN^2}$$

$$= \sqrt{1099^2 + (20 \times 80)^2} = 1941 \text{ V}$$

i) Line current  $I_L =$  phase current  $I$

$$= \frac{E_R}{Z_s} = \frac{1941}{20} = 97.05 \text{ A}$$

ii) p.f  $\cos \phi = \frac{I \cos \phi}{I} = \frac{80}{97.05} = 0.8243$   
(leading)

H.W

A 2000 V, 3 $\phi$ , 4 pole star connected synchronous motor runs at 1500 rpm. The excitation is constant and corresponding to an open ckt voltage of 2000 V. The resistance is negligible in comparison with synchronous reactance of 3.5  $\Omega$ /ph. For an armature current of 200 A, determine (i) p.f (ii) power i/p (iii) torque developed.



## UNIT - III

### THREE PHASE INDUCTION MOTOR

#### Introduction:

The three phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no load to full load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control.

A 3 $\phi$  induction motor has a stator and a rotor. The stator carries a 3 phase winding while the rotor carries a short circuited winding. Only the stator winding is fed from 3 $\phi$  supply. The rotor winding derives its voltage and power from the externally energised stator winding through electromagnetic induction.

The 3-ph induction motors are simple rugged, low-priced, easy to maintain and can be manufactured with characteristics to suit most industrial requirements, but it is essentially a constant speed motor and its speed cannot be changed easily.

## Construction :

A 3 ph induction motor has two main parts (i) stator and (ii) rotor. The rotor is separated from the stator by a small air gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor.

### 1. Stator :

It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. The 3-ph stator winding is wound for a definite number of poles as per requirement of speed. When 3-ph supply is given to the stator winding, a rotating magnetic field of constant magnitude is produced. This rotating field induces currents in the rotor by electromagnetic induction.

### 2. Rotor

The rotor mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots may be one of the following two types. (i) squirrel cage type  
(ii) wound type.

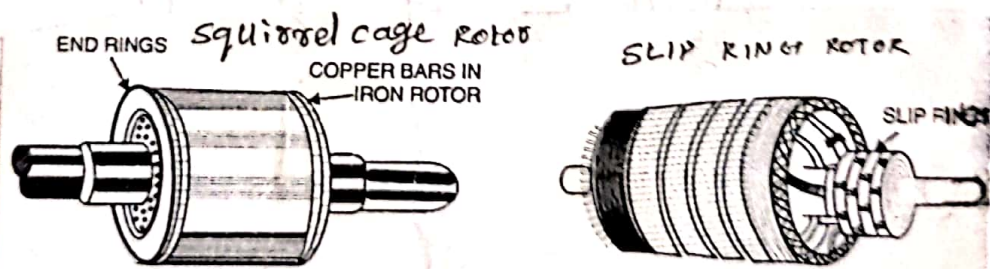


## (i) Squirrel cage rotor:

It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminium bar is placed in each slot. All these bars are jointed at each end by metal rings called end rings.

Most of 3-ph induction motors use squirrel cage rotor as it has robust construction enabling it to operate in the most adverse circumstances.

However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque.



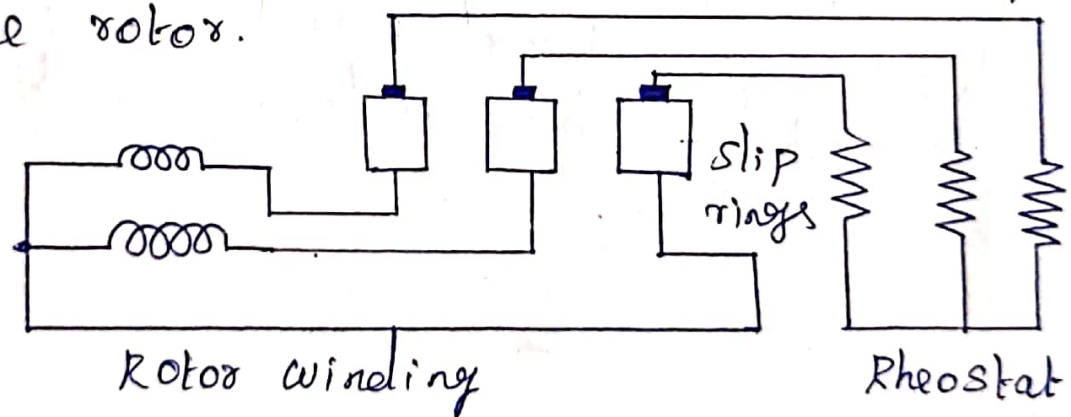
## (ii) Wound rotor:

It consists of a laminated cylindrical core and carries a 3-ph winding similar to the one on the stator. The rotor winding is uniformly distributed in the slots and is usually star connected. The open ends of

The rotor windings are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring.

At starting, the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed.

The external resistances are used during starting period only. When the motor attains normal speed, the three brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.



Rotating magnetic field due to 3 phase current.

When a 3-ph winding is energized from a 3-ph supply, a rotating magnetic field is produced. This field is such that



its poles do not remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating field. (3)

It can be shown that magnitude of this rotating field is constant and is equal to  $1.5 \phi_m$  where  $\phi_m$  is the maximum flux due to ~~any~~ <sup>any</sup> phase.

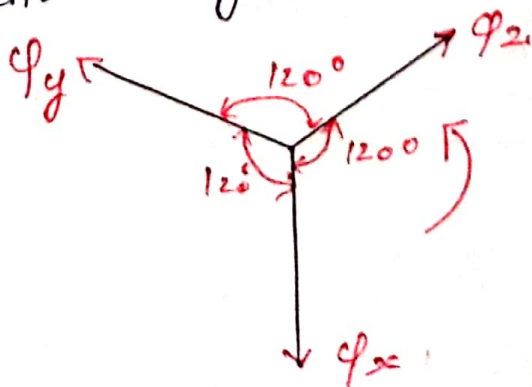
The 3 phases x, y and z are energised from a 3-phase source and currents in these phases are indicated as  $I_x, I_y$  and  $I_z$ . The fluxes produced by these currents are given by,

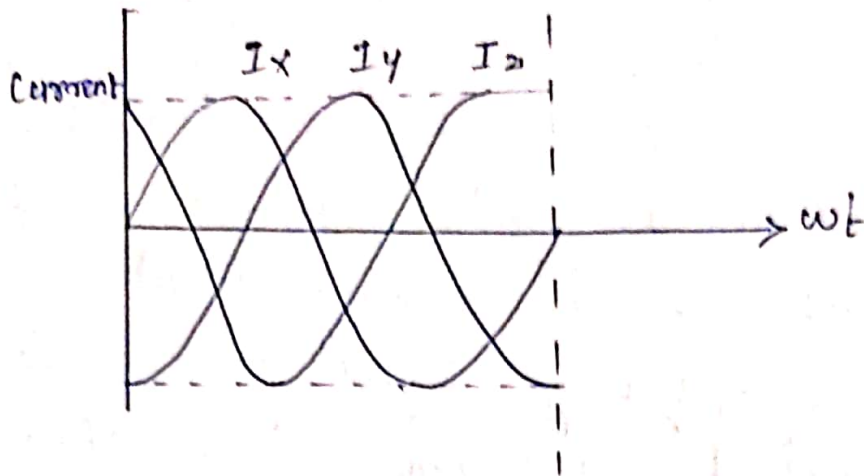
$$\phi_x = \phi_m \sin \omega t$$

$$\phi_y = \phi_m \sin (\omega t - 120^\circ)$$

$$\phi_z = \phi_m \sin (\omega t - 240^\circ)$$

We shall now prove that this 3-ph supply produces a rotating field of constant magnitude equal to  $1.5 \phi_m$ .





at instant 1,  $\omega t = 0^\circ$ , therefore the three fluxes are given by

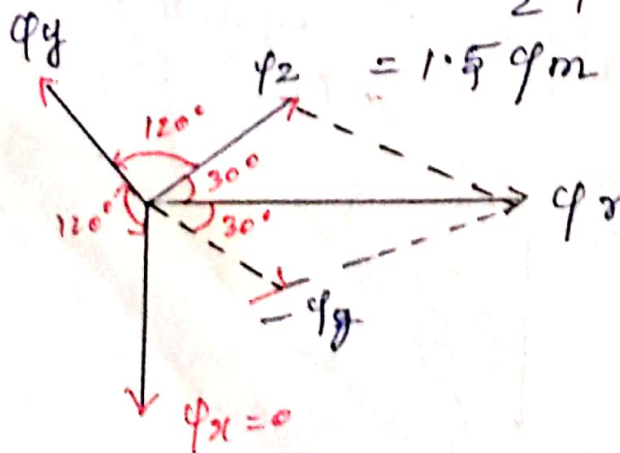
$$\phi_x = 0; \quad \phi_y = \phi_m \sin(-120^\circ) = -\frac{\sqrt{3}}{2} \phi_m;$$

$$\phi_z = \phi_m \sin(-240^\circ) = \frac{\sqrt{3}}{2} \phi_m$$

Resultant flux,  $\phi_r = 2 \times \frac{\sqrt{3}}{2} \phi_m \cos \frac{60^\circ}{2}$

$$= 2 \frac{\sqrt{3}}{2} \phi_m \times \frac{\sqrt{3}}{2}$$

$$= 1.5 \phi_m$$





ii) At instant 2, the current is  $\textcircled{4}$  maximum in phase  $y$  and 0.5 maximum in phases  $x$  and  $z$ .

The magnitude of resultant flux is  $1.5\phi_m$  as proved under: at instant 2, the fluxes are given by

$$\phi_x = \phi_m \sin 30^\circ = \phi_m/2$$

$$\phi_y = \phi_m \sin(-90^\circ) = -\phi_m$$

$$\phi_z = \phi_m \sin(-210^\circ) = \phi_m/2$$

The phasor sum of  $\phi_x$ ,  $-\phi_y$  and  $\phi_z$  is the resultant flux  $\phi_r$

$$\text{phasor sum of } \phi_x \text{ and } \phi_z, \phi_r' = 2 \times \frac{\phi_m}{2} \cos 120^\circ = \phi_m/2$$

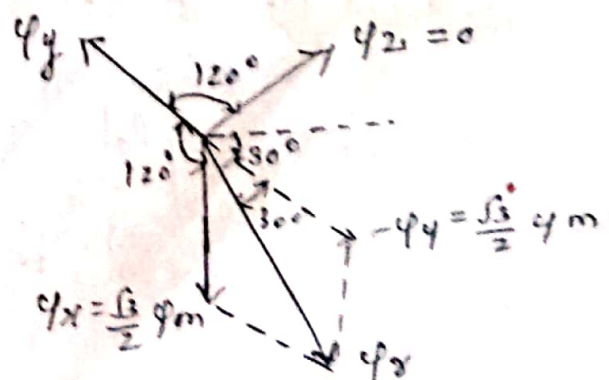
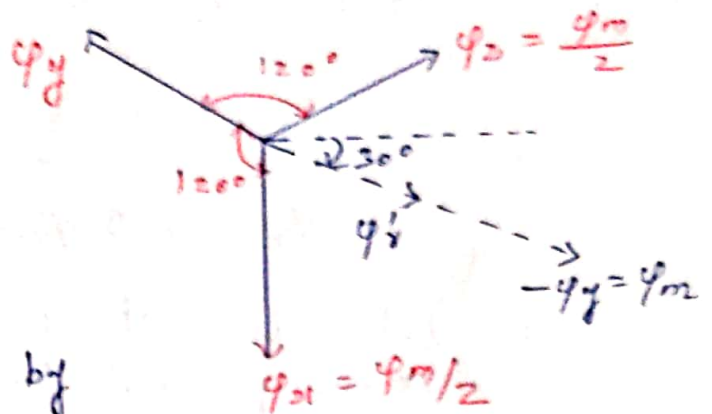
$$\text{phasor sum of } \phi_r' \text{ and } -\phi_y, \phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\phi_m$$

Note that resultant flux is displaced  $30^\circ$  clockwise from position 1.

(iii) At instant 3; current in phase  $z$  is zero and the currents in phases  $x$  and  $y$  are equal and opposite. The magnitude of resultant flux is  $1.5\phi_m$  as proved under.

At instant 3,  $\omega t = 60^\circ$  therefore the 3 fluxes are given by,

$$\begin{aligned} \phi_x &= \phi_m \sin 60^\circ \\ &= \frac{\sqrt{3}}{2} \phi_m \end{aligned}$$



$$\phi_y = \phi_m \sin(-60^\circ) = -\frac{\sqrt{3}}{2} \phi_m$$

$$\phi_z = \phi_m \sin(-180^\circ) = 0$$

$$\therefore \phi_r = 2 \times \frac{\sqrt{3}}{2} \phi_m \cos \frac{60^\circ}{2}$$

$$\phi_r = 1.5 \phi_m$$

iv) At instant 4, the current in phase X is maximum and the current in phases Y and Z are equal and negative.

at instant 4,  $\omega t = 90^\circ$ , therefore the fluxes are given by

$$\phi_x = \phi_m \sin 90^\circ = \phi_m$$

$$\phi_y = \phi_m \sin(-30^\circ) = -\phi_m/2$$

$$\phi_z = \phi_m \sin(-150^\circ) = -\phi_m/2$$

phasor sum of  $-\phi_z$  and  $-\phi_y$

$$\phi_r' = 2 \times \phi_m/2 \cos \frac{120^\circ}{2} = \phi_m/2$$

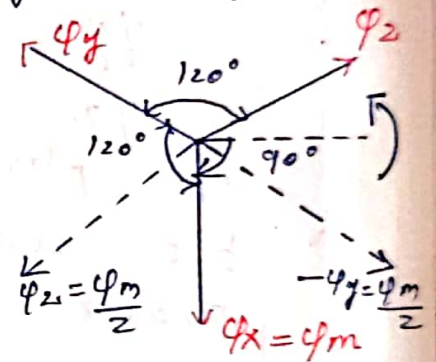
phasor sum of  $\phi_r'$  and  $\phi_x$

$$\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m$$

Note that the resultant flux is downward  $\alpha$ , it is displaced  $90^\circ$  clockwise from position 1.

Speed of rotating magnetic field:

The speed at which the rotating magnetic field revolves is called the synchronous speed. In general, for P poles the rotating field makes one revolution in  $P/2$

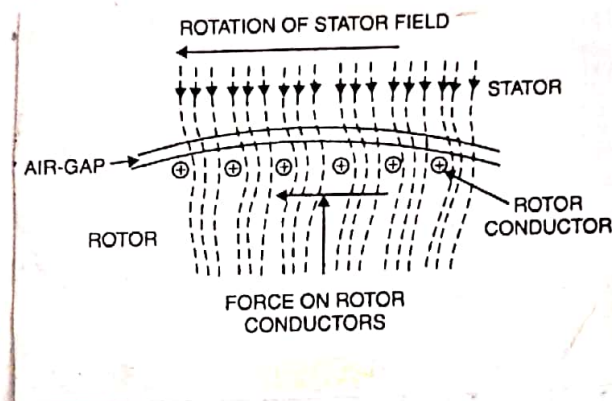




## PRINCIPLE OF OPERATION

Consider a portion of 3-phase induction motor as shown in fig.

(i) when 3-phase stator winding is energised from a 3-phase supply, a rotating magnetic field is set up which rotates around the stator at synchronous speed  $N_s = \frac{120f}{P}$ .



(ii) The rotating field passes through the air gap and cuts the rotor conductors, which as yet are stationary. Due to the relative speed between the rotating flux and the stationary rotor e.m.f.s are induced in the rotor conductors. Since the rotor circuit is short circuited, current starts flowing in the rotor conductors.

(iii) The current carrying rotor conductors are placed in the magnetic field produced by the stator. Consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which

tends to move the rotor in the same directions as the rotating field.

(iv) The fact that rotor is urged to follow the stator field. According to Lenz's law, the direction of rotor current will be such that they tend to oppose the cause producing them.

Now, the cause producing the rotor currents is the relative speed between the rotating field and the stationary rotor conductors. Hence to reduce this relative speed, the rotor starts running in the same direction as that of stator field and tries to catch it.

### SLIP

The difference between the synchronous speed  $N_s$  of the rotating stator field and the actual rotor speed  $N$  is called slip. It is usually expressed as a percentage of synchronous speed.

$$\text{ie } \% \text{ slip, } s = \frac{N_s - N}{N_s} \times 100$$

$N_s - N$  is called slip speed.

$$\text{If } N = 0, s = 1$$



Example

A 3 phase, 50 Hz induction motor runs at 960 rpm on full load. Find the number of poles and slip speed.

Given

$$F = 50 \text{ Hz, speed } N = 960 \text{ rpm}$$

Solution:-

$$N_s = \frac{120f}{P}$$

$$N_s = \frac{120 \times 50}{P}$$

1000 rpm is the nearest synchronous speed of 960 rpm.

$$P = \frac{120 \times 50}{1000} = 6$$

$$P = 6$$

$$\text{slip speed} = N_s - N = 1000 - 960$$

$$\text{slip speed} = 40 \text{ rpm}$$

Examples

A 12 pole 3 phase alternator driven at a speed of 500 rpm, supplies power to an 8 pole, 3-phase induction motor. If the slip of the motor, at full load is 3%. Calculate the full load speed of the motor.

Given data

$$\text{Alternator } P = 12, N = 500 \text{ rpm}$$

$$\text{motor } P = 8, S = 0.03$$

Solution

supply frequency of the motor  $f = \frac{NP}{120}$

$$= \frac{500 \times 12}{120} = 50 \text{ Hz.}$$

synchronous speed of the motor  $N_s = \frac{120f}{P}$

$$= \frac{120 \times 50}{8}$$
$$= 750 \text{ rpm}$$

Motor speed  $N = N_s (1 - S)$

$$= 750 (1 - 0.03)$$

$$= 727.5 \text{ rpm.}$$

$$\boxed{N = 727.5} \text{ rpm}$$

Example: A six pole 134, 60 Hz induction motor runs at 4% slip at a certain load. Determine the synchronous speed, frequency of the rotor current, speed of the rotor rotating field with respect to the stator, and speed of rotor rotating field with respect to the stator magnetic field.



Given data,

Number of poles  $P = 6,$

supply frequency  $f = 60 \text{ Hz},$

slip  $s = 0.04$

Solution:-

(i) synchronous speed  $N_s = \frac{120f}{P}$   
 $= \frac{120 \times 60}{6}$   
 $= 1200 \text{ rpm}$   
 $N_s = 1200 \text{ rpm}$

(ii) Frequency of rotor current  
 $f_r = sf = 0.04 \times 60$   
 $f_r = 2.4 \text{ Hz}$

(iii) speed of the rotor rotating field with respect to the stator.  
 speed of the rotor field with respect to the rotor,  $= 60 \text{ rpm}.$   
 speed of the rotor with respect to stator  $= 1140 \text{ rpm}.$

Thus the speed of the rotor field with respect to stator  $= 60 + 1140$   
 $= 1200 \text{ rpm}.$

ie The rotor field rotates at synchronous speed with respect to stator.

iv) Speed of the rotor rotating field with respect to stator magnetic field.

Speed of the rotor field with respect to stator = 1200 rpm.

Speed of the stator field with respect to stator = 1200 rpm.

Thus the speed of the rotor field with respect to stator field =  $1200 - 1200$   
= 0 rpm.

ie, Rotor field is stationary with respect to stator field.

Frequency of Rotor Current OR EMF.

When the rotor is stationary, the relative speed between the rotor winding and the rotating magnetic field is  $N_s$ .

$$\text{Relative speed} = N_s - N$$

$$\text{Rotor frequency } f_r = \frac{N_s - N}{120/p}$$

$$\text{slip } s = \frac{N_s - N}{N_s}$$



(3)

$$N_s - N = s N_s = s \times \frac{120f}{P}$$

Substituting  $N_s - N = s N_s$

$$= s \times \frac{120f}{P} \text{ in } f_r.$$

$$f_r = s \times \frac{120f}{P} \times \frac{P}{120}$$

$$f_r = sf$$

rotor frequency is also called the slip frequency.

Rotor EMF

Under stand still condition slip  $s=1$ ,

$E_2$  = rotor induced emf per phase under stand still condition.

$E_{2r}$  = rotor induced emf per phase under running condition.

$$\therefore E_2 \propto N_s ; E_{2r} \propto N_s - N$$

Dividing these two equations,

$$\frac{E_{2r}}{E_2} = \frac{N_s - N}{N_s} = s$$

$$E_{2r} = s E_2$$

Rotor current and power Factor.

$R_2$  = Rotor resistance per phase under standstill.

$X_2$  = Rotor reactance per phase under standstill.

$$X_2 = 2\pi f L_2 \text{ } \Omega/\text{ph.}$$

$L_2$  = Inductance / phase of the rotor.

$$Z_{2z} = \sqrt{R_2^2 + X_2^2}$$

$$\text{Rotor current } I_2 = \frac{E_2}{Z_{2z}}$$

$$\text{Rotor power factor } \cos \phi_2 = \frac{R_2}{Z_{2z}}$$

Now in running condition  $f_r = sf$

$$\text{Rotor induced emf } E_{2r} = s E_2$$

Rotor reactance per phase under running condition  $X_{2r} = s X_2 = 2\pi f_r L_2$ .

Rotor impedance per phase under running condition  $Z_{2r} = \sqrt{R_2^2 + (s X_2)^2}$

Rotor current per phase under running condition  $I_{2r} = \frac{s E_2}{Z_{2r}}$

$$= \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}}$$



Rotor power factor under running

$$\text{Condition } \cos \phi_{2r} = \frac{R_2}{Z_{2r}}$$
$$= \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Example A 3 phase 16 pole, 50 Hz induction motor has a slip of 1% at no load and 3% at full load. Find (1) synchronous speed (2) no load speed (3) full load speed (4) frequency of rotor current at standstill (5) frequency of rotor current at full load.

Given data:

$p = 6$ ,  $f = 50 \text{ Hz}$ , slip at no load  $s_{nl} = 1\%$   
or  $0.01$ , slip at full load  $s_{fl} = 3\%$  or  $0.03$ .

Solution:-

1) synchronous speed  $N_s = \frac{120f}{p} = \frac{120 \times 50}{6}$   
 $= 1000 \text{ rpm.}$

$$N_s = 1000 \text{ rpm}$$

2) No load speed ( $N_{nl}$ )

$$N_{nl} = N_s(1 - s_{nl})$$
$$= 1000(1 - 0.01) = 990 \text{ rpm.}$$

$$N_{nl} = 990 \text{ rpm}$$

3) Full load speed ( $N_{fl}$ )

$$N_{fl} = N_s(1 - s_{fl})$$
$$= 1000(1 - 0.03)$$

$$N_{fl} = 970 \text{ rpm}$$

4) Frequency of rotor current at standstill ( $f_{rs}$ )

$$f_{rs} = sf$$

At stand still slip  $s = 1$

$$f_{rs} = f = 50 \text{ Hz}$$

5) Frequency of rotor current at full load  $f_{rfl}$

$$f_{rfl} = s_{fl}f = 0.03 \times 50 = 1.5 \text{ Hz}$$

$$f_{rfl} = 1.5 \text{ Hz}$$



Example

(5)  
A 1100V, 50 Hz delta connected induction motor has a star-connected slip ring rotor with a phase transformation ratio of 3.8. The rotor resistance and stand still leakage reactance are  $0.012\Omega$  and  $0.25\Omega$  per phase respectively. Neglecting stator impedance and magnetising current, determine.

- (i) Rotor current at start with slip ring shorted.
- (ii) The rotor p.f at start with slip ring shorted.
- (iii) The rotor current at 4% slip with slip ring shorted.
- (iv) The rotor power factor at 4% slip with slip ring shorted.
- (v) The external rotor resistance per phase required to obtain a starting current of 100 A in the stator supply lines.

Given data:

supply voltage  $V = 1100\text{V}$

supply frequency  $f = 50\text{Hz}$ .

phase transformation ratio  $k = \frac{1}{3.8} = 0.263$ .

Rotor resistance per phase  $R_2 = 0.012 \Omega$

Rotor reactance per phase  $X_2 = 0.25 \Omega$

Soln:-

Rotor phase voltage at stand still

$$E_{2ph} = 1100 \times 0.263 \\ = 289.3 \text{ V}$$

(i) Rotor current at starting condition,

$$I_2 = \frac{E_{2ph}}{Z_2} = \frac{289.3}{\sqrt{R_2^2 + X_2^2}} = \frac{289.3}{\sqrt{0.012^2 + 0.25^2}}$$

$$I_2 = 1157.2 \text{ A}$$

(ii) Rotor power factor at starting condition,

$$\cos \phi = \frac{R_2}{Z_2} = \frac{0.012}{\sqrt{0.012^2 + 0.25^2}}$$

$$\cos \phi_2 = 0.048 \text{ (lagging)}$$

(iii) Rotor current at 4% slip,

$$I_{2s} = \frac{sE_2}{Z_{2s}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \\ = \frac{0.04 \times 289.3}{\sqrt{0.012^2 + (0.04 \times 0.25)^2}}$$



$$\boxed{I_{2s} = 742} \text{ A}$$

iv) Rotor power factor at 4% slip.

$$\cos \phi_{2s} = \frac{R_2}{Z_{2s}} = \frac{0.012}{\sqrt{0.012^2 + (6.04 \times 0.25)^2}}$$

$$\cos \phi_{2s} = 0.77 \text{ (lagging)}$$

(v) The external rotor resistance per phase

$$I_2 = \frac{I_1}{k} = \frac{100}{0.263} = 380.2 \text{ A.}$$

$$E_2 = 289.3 \text{ V}$$

$$Z_2 = \frac{E_2}{I_2} = \frac{289.3}{380.2} = 0.761 \Omega$$

$$R_2 = \sqrt{Z_2^2 - X_2^2} = \sqrt{0.761^2 - 0.25^2}$$

$$= 0.719 \Omega$$

∴ External resistance required / phase

$$r = 0.719 - 0.012$$

$$\boxed{r = 0.707} \Omega$$

## Torque Equation:-

In DC motor, the torque  $T$  is proportional to the product of the armature current and flux per pole  
i.e.  $T \propto \phi I_a$ .

In the case of the induction motor, however, in addition to the flux and rotor current, the rotor power factor has also to be taken in to account.

$$\text{Hence, } T \propto \phi I_{2r} \cos \phi_{2r} \quad \text{①}$$

where

$\phi$  = flux responsible to produce induced emf

$I_{2r}$  = rotor current under running condition.

$\cos \phi_{2r}$  = rotor power factor under running condition.

Let  $E_2 \rightarrow$  rotor induced emf per phase under standstill condition,

$X_2 \rightarrow$  Rotor reactance per phase under standstill condition.

rotor frequency at slip  $s$  is

$$f_r = sf.$$



The rotor reactance varies,

$$X_{2r} = s X_2.$$

Also  $E_{2r} \propto \phi$

$$E_{2r} = s E_2$$

$$\text{and } I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} \quad \text{--- (2)}$$

$$\cos \phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (s X_2)^2}} \quad \text{--- (3)}$$

$\phi$  can be replaced by  $E_2$   
sub (2) and (3) in (1)

$$T \propto E_2 \cdot \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (s X_2)^2}}$$

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

$$T = \frac{k s E_2^2 R_2}{R_2^2 + (s X_2)^2} \quad \text{N-m} \quad \text{--- (4)}$$

$$k = \frac{3}{2\pi n_s}$$

$$n_s = \frac{N_s}{60} \Rightarrow \text{synchronous speed in rps.}$$

At standstill,  $s = 1$  and therefore the

$$\text{starting torque, } T_{st} = \frac{k E_2^2 R_2}{R_2^2 + X_2^2} \quad \text{N-m}$$

--- (5)

Condition for maximum running torque.

Torque under running condition,

$$T = \frac{s k E_2^2 R_2}{R_2^2 + (s X_2)^2} \quad N-m$$

Torque  $T$  for a fixed input voltage will be maximum when  $\frac{dT}{ds} = 0$ ,

$$\therefore \left( R_2^2 + s^2 X_2^2 \right) \left( k E_2^2 R_2 \right) - \left( k s E_2^2 R_2 \right) \left( 2 s X_2^2 \right) = 0 \quad \text{--- (6)}$$

$$\text{i.e. } R_2 = s X_2$$

$s_m = \frac{R_2}{X_2}$  is the slip at which

the torque is maximum.

Then substituting  $s = \frac{R_2}{X_2}$  in eqn (4)

$$\text{we have, } T_{\max} = \frac{k s E_2^2 s X_2}{2 s^2 X_2^2}$$

$$T_{\max} = \frac{k E_2^2}{2 X_2} \quad \text{--- (7)}$$

From this eqn, it can be observed that,

1. The maximum torque is independent of rotor resistance  $R_2$ .



2. Maximum torque is directly proportional to the square of the induced emf at standstill.  $\propto (E_2^2)$
3. Maximum torque is inversely proportional to the rotor reactance.

Starting Torque and maximum torque.

$$T_{st} \propto \frac{R_2}{R_2^2 + X_2^2}$$

$$T_{max} \propto \frac{1}{2X_2}$$

$$\frac{T_{st}}{T_{max}} = \frac{2R_2 X_2}{R_2^2 + X_2^2} = \frac{2R_2 / X_2}{1 + (R_2 / X_2)^2}$$

$$\frac{T_{st}}{T_{max}} = \frac{2a}{1+a^2}$$

where  $a = \frac{R_2}{X_2}$

Full load torque and maximum torque

$$T_{fl} \propto \frac{s_f R_2}{R_2^2 + (s_f X_2)^2}$$

$$T_{max} \propto \frac{1}{2X_2}$$

$$\frac{T_{fl}}{T_{max}} = \frac{2s_f R_2 X_2}{R_2^2 + (s_f X_2)^2}$$

dividing both the numerator and

the denominator by  $x_2^2$ ,

$$\begin{aligned} \text{we can get } \frac{T_{fl}}{T_{max}} &= \frac{2s_f R_2 / x_2}{(R_2 / x_2)^2 + s_f^2} \\ &= \frac{2a s_f}{a^2 + s_f^2} \end{aligned}$$

$$\frac{T_{fl}}{T_{max}} = \frac{2a s_f}{a^2 + s_f^2}$$

$$\text{where } a = \frac{R_2}{x_2}$$

Effect of change in supply voltage :-

The torque equation of the induction motor is given by,

$$T = \frac{k\phi s E_2^2 R_2}{R_2^2 + (s x_2)^2}$$

Here,  $E_2 \propto \phi \propto V$  (supply voltage)

Torque at any speed is proportional to the square of the supply voltage.

$$\text{i.e. } T \propto V^2$$

→ Changing the supply voltage, affects the starting torque, maximum torque and torque under running condition.



Let  $V$  changes to  $V_1$ ,  $s \leq s_1$ .

$$T = sV^2, \quad T_1 = s_1V_1^2$$

$$\boxed{\frac{T}{T_1} = \frac{sV^2}{s_1V_1^2}}$$

Example:-

A 3300V, 10 pole 150 Hz 134 star connected induction motor has a slip ring rotor resistance per phase =  $0.015 \Omega$  and a standstill reactance per phase =  $0.25 \Omega$ . If the motor runs at 2.5 percent slip on full load find (i) the speed of the motor (ii) speed at which the torque will be maximum (iii) the ratio of maximum torque to full load torque.

Given data:-

$$V = 3300 \text{ V}, \quad p = 10, \quad f = 50 \text{ Hz}$$

$$R_2 = 0.015 \Omega, \quad X_2 = 0.25 \Omega$$

$$s = 2.5\% = 0.025$$

solution:-

i) The speed of the motor  $N$

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{10} = 600 \text{ rpm.}$$

$$\text{Motor speed } N = N_s(1-s) = 600(1-0.025)$$

$$N = 585 \text{ rpm.}$$

ii) speed at which the torque will be maximum,

$$s_m = \frac{R_2}{X_2}$$

$$s_m = \frac{0.015}{0.25} = 0.06$$

$$N = N_s(1 - s_m) = 600(1 - 0.06)$$

$$N = 564 \text{ rpm}$$

(iii) The ratio of maximum torque to full load torque

$$T_{\max} \propto \frac{1}{2X_2}$$

$$T_f \propto \frac{s_f R_2}{R_2^2 + (s_f X_2)^2}$$

$$\begin{aligned} \frac{T_{\max}}{T_f} &= \frac{\frac{1}{2X_2}}{\frac{s_f R_2}{R_2^2 + (s_f X_2)^2}} = \frac{1}{2X_2} \times \frac{R_2^2 + (s_f X_2)^2}{s_f R_2} \\ &= \frac{1}{2 \times 0.25} \times \frac{0.015^2 + (0.025 \times 0.25)^2}{0.025 \times 0.015} \end{aligned}$$

$$\frac{T_{\max}}{T_f} = 1.408$$



ex) A 4 pole, 50 Hz, 7.46 kW motor has, at rated voltage and frequency, a starting torque of 160 percent and a maximum torque of 200 percent of full load torque. Determine (1) full load speed (2) speed at maximum torque.

Given data :-

$$P = 4, f = 50 \text{ Hz}, T_{st} = 1.6 T_{fl}, T_{max} = 2 T_{fl}$$

soln :-

$$\frac{T_{st}}{T_{max}} = \frac{1.6}{2} = 0.8$$

$$T_{max} = 2 T_{fl}$$

$$\therefore \frac{T_{max}}{T_{fl}} = 2$$

$$\frac{T_{st}}{T_{max}} = \frac{2a}{1+a^2}$$

$$\therefore \frac{2a}{1+a^2} = 0.8$$

$$0.8a^2 - 2a + 0.8 = 0$$

Solving this equation, we can get

$$a = 0.5$$

$$a = \frac{R_2}{X_2} = 0.5$$

$$R_2 = 0.5 X_2$$

$$\frac{T_{fl}}{T_{max}} = \frac{1}{2} = \frac{2asf}{a^2 + sf^2}$$

$$0.5 = \frac{2 \times 0.5 \times s_f}{(0.5)^2 + s_f^2}$$

$$(0.5)(0.5^2 + s_f^2) = 2 \times 0.5 \times s_f$$

$$0.125 + 0.5 s_f^2 = s_f$$

$$0.5 s_f^2 - s_f + 0.125 = 0$$

$$s_f = 0.13$$

i) Full load speed occurs at a slip of 0.01.

$$\text{Synchronous speed } N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

$$\begin{aligned} \therefore \text{Motor speed } N &= N_s (1 - s) \\ &= 1500 (1 - 0.13) \\ &= 1305 \text{ rpm} \end{aligned}$$

$$N = 1305 \text{ rpm}$$

(ii) Maximum torque occurs at a slip of

$$s_m = \frac{R_2}{X_2} = 0.5$$

$$\begin{aligned} N &= N_s (1 - s) \\ &= 1500 (1 - 0.5) \end{aligned}$$

$$N = 750 \text{ rpm}$$



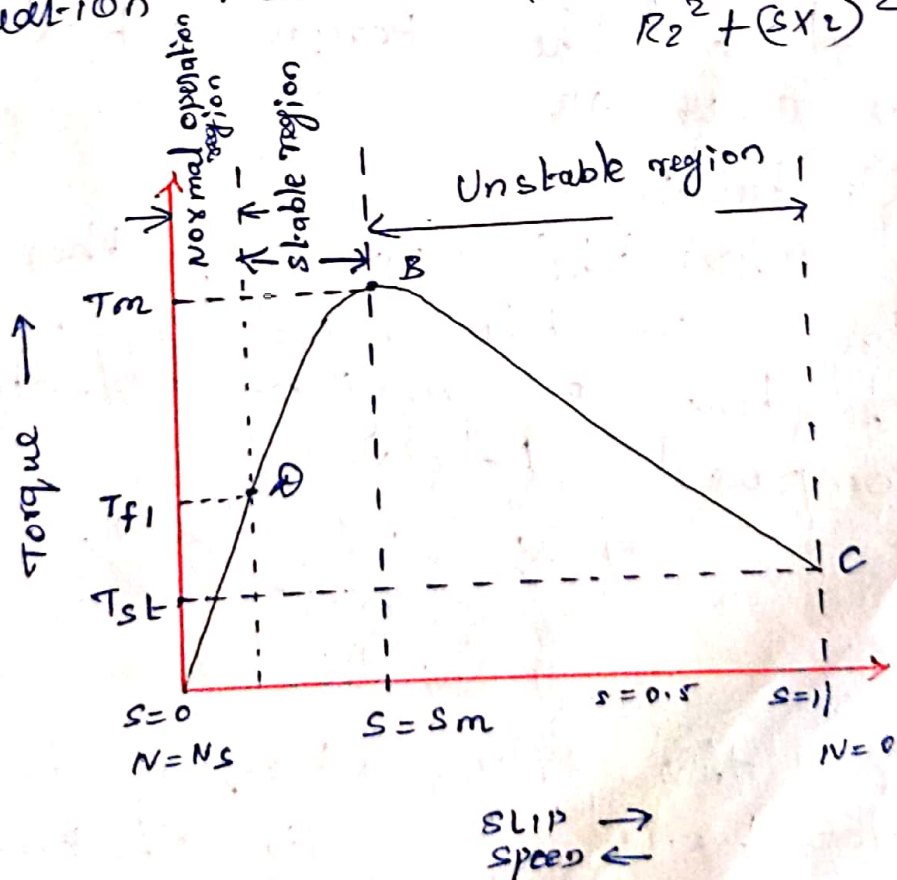
## TORQUE - SLIP CHARACTERISTICS.

The curve drawn between torque and slip from  $s=1$  to  $s=0$  is called torque - slip characteristics of the induction motor.

The torque equation of induction motor is given by

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

Here, the i/p voltage is constant, i.e.  $E_2$  is also constant. So the above equation becomes,  $T \propto \frac{s R_2}{R_2^2 + (s X_2)^2}$



Torque slip characteristics are 2 regions

- 1) stable region
- 2) Unstable operating region
- 3) Normal operating region.

stable region

In stable region, the slip value  $s$  is very small i.e. the term  $(sX_2)^2$  is very small as compared to  $R_2^2$ . Hence neglecting  $s^2 X_2^2$ .

$$T \propto \frac{s R_2}{R_2^2} \propto s \text{ as } R_2 \text{ is constant.}$$

$\therefore T \propto s$

The slip value is directly proportional to the torque. It is indicated in curve AB.

Unstable region.

When the slip is further increased from  $s_m$ , the region is unstable region. The term  $R_2^2$  may be neglected as compared to  $s^2 X_2^2$ .

$$T \propto \frac{s}{(sX_2)^2} \propto \frac{1}{s}$$

$\therefore T \propto \frac{1}{s}$

In this region, torque is inversely proportional to slip. The torque slip curve is similar to a rectangular hyperbola.



Normal operating region

The motor is continuously operated in this region. The following terms,

1. Starting torque ( $T_{st}$ )

2. Maximum torque or pull out torque ( $T_m$ )

3. Full load torque ( $T_{FL}$ )

Starting torque: In torque slip characteristics when the slip is zero. At this condition the motor produces a torque called starting torque.

Maximum torque:

When slip  $s = s_m$  is called maximum torque, it is also called breakdown torque or pull out torque.

Full load torque:

Normally full load torque is less than the maximum torque.

LOSSES IN AN INDUCTION MOTOR.

Magnetic losses:

→ The magnetic losses are also called core losses or iron losses. These losses occur in the stator core and

rotor core.

→ The hysteresis losses due to changes in the magnetic field in the stator core.

→ The eddy current losses are due to the flow of eddy current through the body of the stator core.

Mechanical loss:

Frictional and windage losses are mechanical loss.

Electrical losses:

These losses are due to the resistance of stator and rotor winding. These losses includes stator copper loss and rotor copper loss. These are variable losses.

$$\text{Rotor Cu loss} = 3I_2^2 R_2$$

Power Flow

Three phase supply is fed to the stator of the induction motor. The i/p power is  $P_{in} = \sqrt{3} V_L I_L \cos \phi$

Some losses occur in the stator.

These losses are called stator losses.

$$P_{sL} = \text{Combination of stator core loss} + \text{Copper loss in stator.}$$



∞ stator o/p or rotor i/p

$$P_2 = P_{in} - P_{sL}$$

$$\text{rotor cu loss } P_{cu} = 3I_2^2 R_2$$

The gross mechanical power

$$P_m = P_2 - P_{cu}$$

The power available to the load at the shaft

$$P_{out} = P_m - P_{mL}$$

$P_{mL}$  → mechanical loss.

$$\text{Rotor efficiency} = \frac{P_m}{P_2}$$

$$\text{Motor efficiency} = \frac{P_{out}}{P_{in}}$$

Ex:

A 100 kW (o/p) 13300 V 13φ, star connected induction motor has a synchronous speed of 500 rpm. The full load slip is 1.8% and full load power factor 0.85. stator copper loss = 2440 W. Iron loss = 3500 W. Rotational losses = 1200 W. calculate (1) the rotor copper loss (2) the line current (3) the full load efficiency.

Given data:  $P_{out} = 100 \text{ kW}$ ,  $V_L = 3300 \text{ V}$

$N_s = 500 \text{ rpm}$ ,  $SF = 1.8\% \text{ or } 0.018$

$\cos\phi = 0.85$ ,  $P_{sL} = 2440 \text{ W}$ ,  $P_i = 3500 \text{ W}$ ,

$P_{mL} = 1200 \text{ W}$ .

Soln:

$$1) P_{out} = 100 \text{ kW.}$$

mechanical power  $P_m$  =  $P_{out}$  +  $P_{mL}$   
developed

$$= 100 + 1.2 = 101.2 \text{ kW}$$

$$P_{cu} = \frac{s}{1-s} \times P_m = \frac{0.018}{1-0.018} \times 101.2$$

$$P_{cu} = 1.855 \text{ kW}$$

2) The Line current ( $I_L$ )

$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

$$P_{in} = P_m + P_{cu} + P_{sL} + \text{Iron loss}$$

$$= 101.2 + 1.855 + 2.44 + 3.5$$

$$P_{in} = 108.995 \text{ kW}$$

$$108.995 \times 10^3 = \sqrt{3} \times 330 \times I_L \times 0.85$$

$$I_L = 22.4 \text{ A}$$

3) Full load efficiency

$$\eta = \frac{P_{out}}{P_{in}} \times 100$$

$$= \frac{100000}{108995} \times 100 = 91.7\%$$

$$\eta = 91.7\%$$



Relationship between rotor input ( $P_2$ )  
rotor copper loss ( $P_{cu}$ ) and gross  
mechanical power ( $P_m$ ).

Let  $T_g$  = Gross torque developed by motor  
in N-m.

The general expression for power is related  
with torque,  $P = T_g \times \omega$

where  $P$  = Power,  $\omega$  = angular speed.

$$P = T_g \times \frac{2\pi N}{60}$$

Power input to the rotor  $P_2$  is given by

$$P_2 = T_g \times \omega_s = T_g \times \frac{2\pi N_s}{60} \quad \text{--- (1)}$$

where  $N_s$  = synchronous speed

Gross mechanical power developed  
by rotor  $P_m$  is given by,

$$P_m = T_g \times \omega = T_g \times \frac{2\pi N}{60}$$

Rotor copper loss

$$P_{cu} = P_2 - P_m = T_g \times \frac{2\pi N_s}{60} - T_g \frac{2\pi N}{60}$$

$$P_{cu} = T_g \frac{2\pi}{60} (N_s - N) \quad \text{--- (2)}$$

dividing the eqn (2) by (1)

$$\frac{P_{cu}}{P_2} = \frac{T_g \frac{2\pi}{60} (N_s - N)}{T_g \times \frac{2\pi N_s}{60}}$$

$$= \frac{N_s - N}{N} = s$$

∴ Rotor copper loss,  $P_{cu} = s \times \text{rotor i/p, } P_2$

$$\text{(or)} \quad P_2 = \frac{P_{cu}}{s} \quad \text{--- (3)}$$

Gross mechanical power developed by rotor  $P_m = \text{rotor i/p} - \text{rotor copper loss}$

$$\text{i.e. } P_m = P_2 - P_{cu} = P_2 - sP_2$$

$$\therefore P_m = P_2 (1 - s) \quad \text{--- (4)}$$

From eqn (3) and (4) we get,

$$\frac{\text{rotor copper loss}}{\text{mechanical power developed}} = \frac{sP_2}{(1-s)P_2} = \frac{s}{1-s}$$

The relationship can be expressed as, (5)

Rotor i/p : mechanical power developed by rotor :  
rotor copper loss.

$$P_2 : P_m : P_{cu} = P_2 : P_2 (1 - s) : sP_2$$

$$\frac{P_2}{P_2} : \frac{P_2(1-s)}{P_2} : \frac{sP_2}{P_2}$$



we get  $1 : (1-s) : s = P_2 : P_m : P_{cu}$

Synchronous watt.

The gross torque of an induction motor is given by,

$$T_g = \frac{P_2 \times 60}{2\pi N_s}$$

From above equation

$$T_g \propto P_2.$$

∴ Torque is synchronous watt ( $T_{sw}$ ) =

Rotor input ( $P_2$ )

$$= \frac{\text{Rotor Copper loss}}{s}$$

$$= \frac{\text{Mechanical power developed}}{1-s}$$

Example: An 18.65 kW, 4 pole, 50 Hz, 3φ IM has friction and windage losses of 2.5% of the o/p. Full load slip is 4%. Find for full load (1) rotor copper loss (2) rotor i/p (3) shaft torque (4) the gross electro magnetic torque.

Given data:-

$$P_{out} = 18.65 \text{ kW}$$

$$P = 4, f = 50 \text{ Hz}$$

Friction & windage losses = 2.5% of the

$$\% \text{ i/p} = 0.025 \times 18.65$$

$$= 0.466 \text{ kW}$$

$$\text{Slip} = 4\% \text{ or } 0.04$$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

Mechanical power developed by the motor,

$$P_m = P_{out} + P_{ml}$$

$$= 18.65 + 0.466$$

$$P_m = 19.116 \text{ kW}$$

(i) Rotor copper loss  $P_{cu} = P_m \cdot \frac{s}{1-s}$

$$= 19.116 \times 10^3 \times 0.04$$

$$1 - 0.04$$

$$P_{cu} = 0.796 \text{ kW}$$

(ii) Rotor i/p  $P_2$ ,  $P_2 = P_m + P_{cu}$

$$= 19.116 + 0.796 = 19.912 \text{ kW}$$



$$P_2 = 19.912 \text{ kW}$$

(iii) shaft torque

$$T_{sh} = \frac{P_{out} \times 60}{2\pi N}$$

$$N = N_s (1 - s)$$

$$= 1500 (1 - 0.04) = 1440 \text{ rpm.}$$

$$T_{sh} = \frac{18.65 \times 10^3 \times 60}{2\pi \times 1440} = 126.76 \text{ N-m}$$

$$T_{sh} = 123.76 \text{ Nm}$$

iv) Gross torque  $T_g$

$$T_g = \frac{P_m \times 60}{2\pi N}$$

$$= \frac{19.116 \times 10^3 \times 60}{2\pi \times 1440}$$

$$= 126.76 \text{ Nm}$$

$$T_g = 126.76 \text{ Nm}$$

## UNIT - IV

STARTING AND SPEED CONTROL OF  
THREE PHASE INDUCTION MOTOR.



①

Need for starter:-

When a 3 $\phi$  induction motor is switched on at normal supply voltage, heavy current will flow through the motor because at the time of starting, there is no back emf. This initial inrush of excessive current is objectionable because it will produce large line voltage drop. This will affect the operation of other electrical equipments connected to the same line. Due to this, starters are used for starting the three phase induction motors.

TYPES OF STARTERS :-

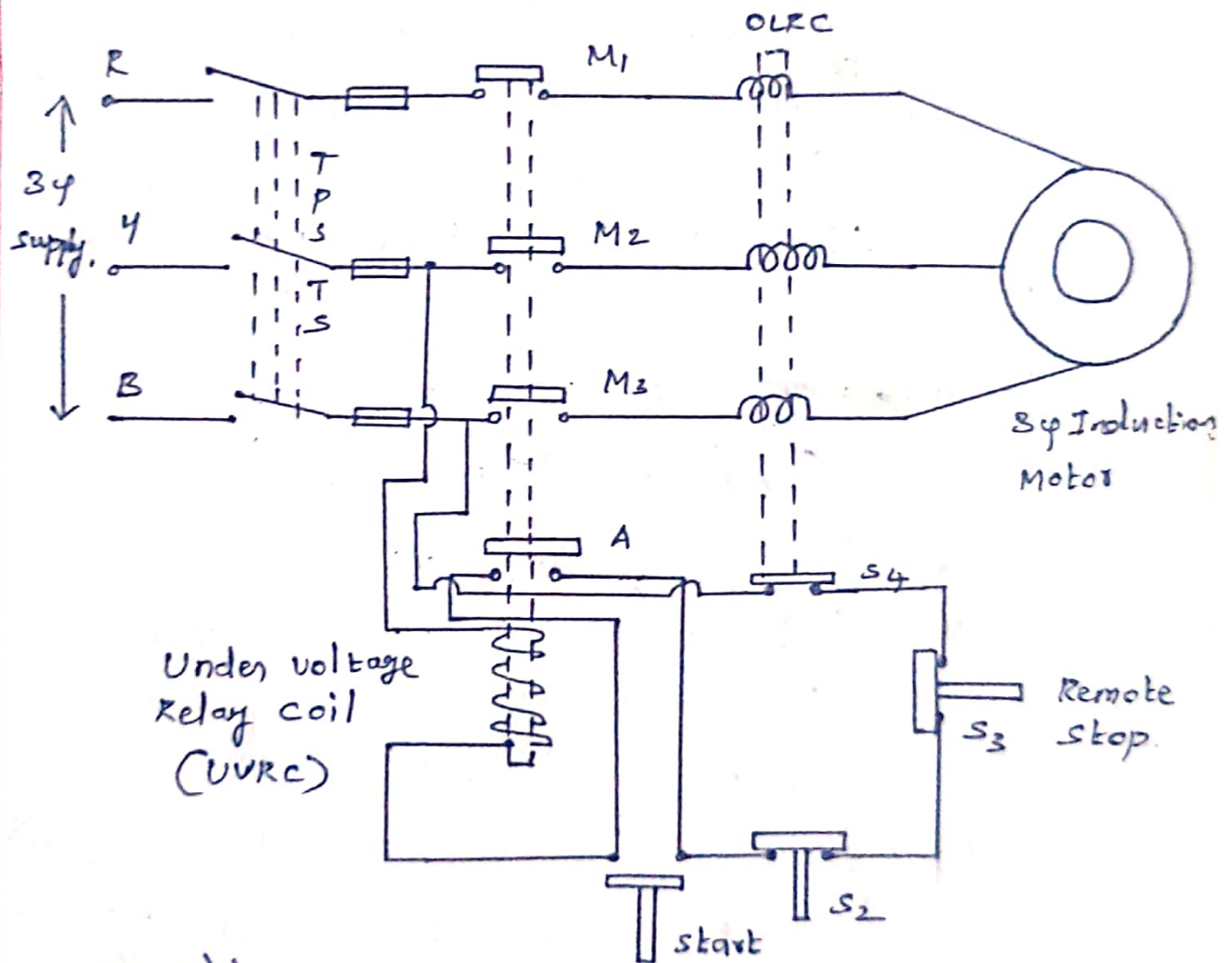
- i) DOL starter
- ii) Primary resistance starter
- iii) Auto transformer starter
- iv) Star-delta starter
- v) Rotor resistance starter

DIRECT ON LINE STARTER (DOL)

A motor of small capacity can be started with this starter. Fig shows a 3 $\phi$  induction motor with a DOL starter.  $M_1, M_2, M_3$  are main

contactors normally open (NO) type making making and breaking the motor line current.

These contactors are operated by a relay coil.  $S_2$ ,  $S_3$  and  $S_4$  are normally closed (NC) type and are connected in series with relay coil. Overload relay coil (OLRC) is connected in series with motor line supply.



operation:-  
 When TPST switch is closed, the under voltage relay coil (UVRC) is energized and it will operate the main contactors



(2)

to close. Hence the full voltage is given to the motor and it runs. Closing of contactor A retains the supply to the UVRC.

Contactors  $S_2$  is used to disconnect the supply from the motor by manually pressing it. Remote operation of the same can be achieved with the help of contactors  $S_3$ .

No voltage protection:

When the supply voltage either fails totally or falls below certain value, the holding power given by UVRC comes down causing the main contactor to be opened. Thus the motor is protected from low voltage operation.

Over load protection:-

When the line current exceeds the preset value, OLRC is energised more and causes the contactors  $S_4$  to open. When  $S_4$  opens, the UVRC is disconnected from the supply. Therefore it will release the main contactors.

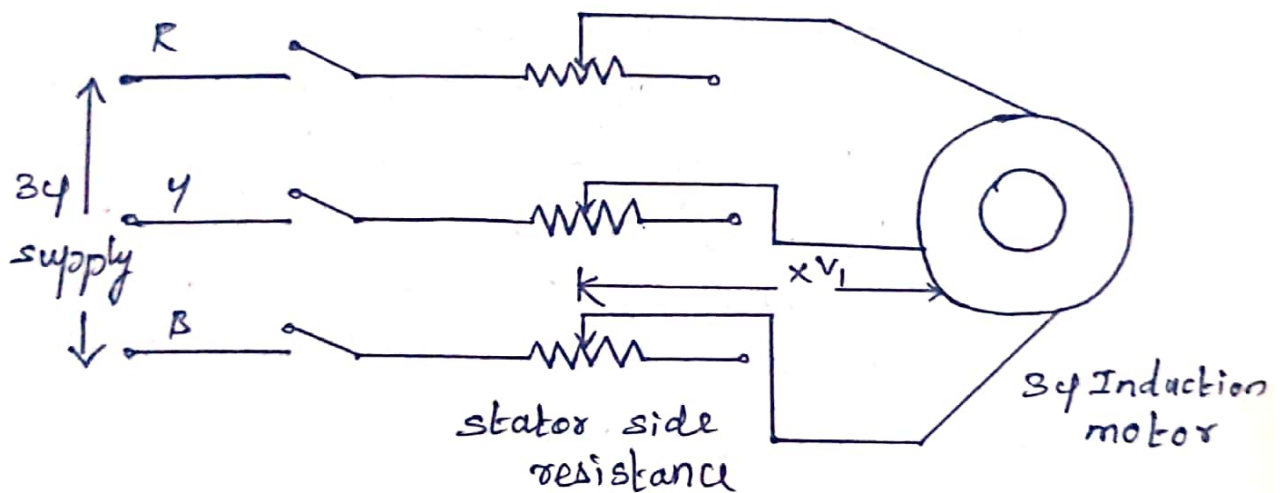
The relation between starting torque  $(T_{st})$  and full load torque  $T_{fl}$  is given by,

$$\frac{T_{st}}{T_{fl}} = \left( \frac{I_{sc}}{I_{fl}} \right)^2 s_f$$

where  $I_{sc} = I_{st}$  = short circuit current  
 $s_f$  = Full load slip.

### Primary Resistor (or) reactor starter.

A variable resistor (or) reactor is connected in series with the supply terminals of the motor. The purpose of resistance is to reduce the supply voltage.



The reduced voltage limits the starting current. If the voltage across the terminal is reduced by 50%, then the starting current is reduced by 50%, but the torque is reduced to 25% of the full voltage value.

Let reduced per phase voltage =  $xV_1$   
 per phase starting current  $I_{st} = \frac{xV_1}{Z_{sc}} = xI_{sc}$

we know that  $\frac{T_{st}}{T_{fl}} = \left(\frac{I_{sc}}{I_{fl}}\right)^2 s_f$



(3)

$$\frac{T_{st}}{T_{fl}} = x^2 \left( \frac{I_{sc}}{I_{fl}} \right)^2 s_f$$

In an induction motor,

$$\text{torque} \propto \text{voltage}^2$$

$$\therefore \frac{\text{starting torque with reactor starting}}{\text{starting torque with direct switching}} = \left[ \frac{xV_1}{V_1} \right]^2 = x^2$$

Advantages :-

1. smooth acceleration
2. High power factor during start
3. Less expensive
4. closed transition starting.

Disadvantages :-

1. Power loss in resistors
2. Low starting torque
3. Less efficiency.

Auto transformer starter :-

This starter is used to give a reduced voltage to the 3-phase induction motor to limit the starting current. The reduced voltage is obtained by an autotransformer.

The supply is given to terminals 1, 3 and 5 of the movable handle and

The motor is connected to 2, 4 and 6 of handle through an OLRC (Overload Release Coil). Low voltage protection is given to the motor by UVRC.

Operation:-

When the handle is at start position, the motor is connected through the auto transformer. When the motor gets 80% of the normal speed, the handle is moved to RUN position. At this position, the motor receives full line voltage.

Overload protection:-

When motor current exceeds the preset value, the over load relay coil is energised high enough to operate the contactor  $S_1$ . Hence supply is switched off.

Low voltage protection:-

The under voltage relay coil is connected across two lines. When supply voltage goes low or fails, UVRC de-energised and releases the handle to OFF position.

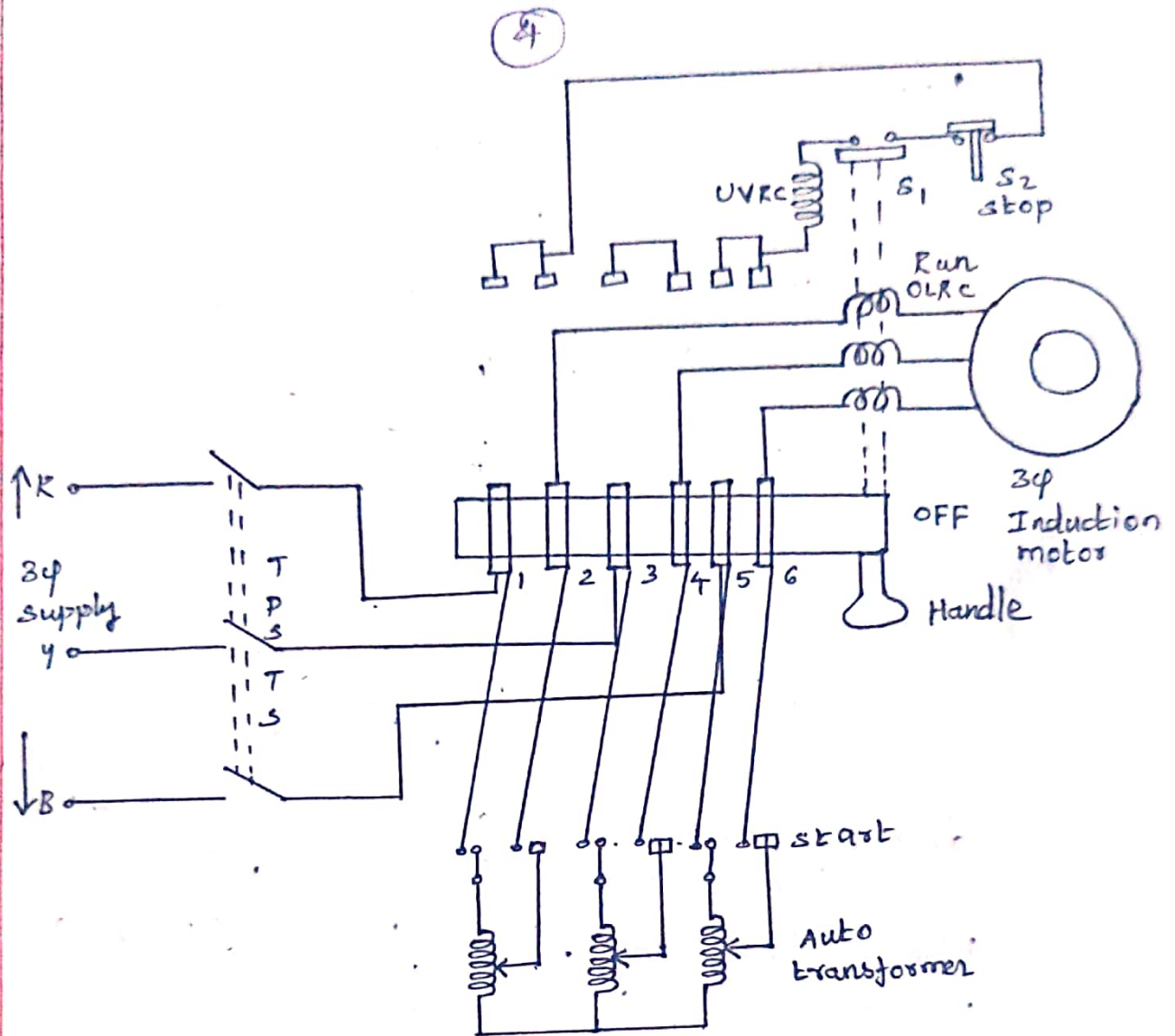
Starting current  $I_{st} = x I_{sc}$

The starting current drawn from the supply  
 $= x(I_{st}) = x(x I_{sc}) = x^2 I_{sc}$

The torque developed by the motor,

$$T = \frac{3 I_2^2 R_2}{s} \quad (\text{synchronous watts})$$





$$\text{Full load torque } T_{fl} = \frac{3I_{fl}^2 R_2}{s_f}$$

$$\text{starting torque } T_{st} = \frac{3I_{st}^2 R_2}{1}$$

[s = 1 at start]

$$\text{Thus } \frac{\text{starting torque}}{\text{Full load torque}} = \frac{T_{st}}{T_{fl}} = \left[ \frac{I_{st}}{I_{fl}} \right]^2 s_f$$

Hence in auto transformer starters,

$$\frac{T_{st}}{T_{fl}} = \left( x \frac{I_{sc}}{I_{fl}} \right)^2 s_f = x^2 \left( \frac{I_{sc}}{I_{fl}} \right)^2 s_f$$

## Advantages:

1. Reduced line current
2. Smooth starting
3. High acceleration

## Disadvantages :-

1. cost is high
2. It is not used for large motors.

## Star delta starter:-

This method is used in motors which are meant to run normally with a delta connected stator winding. It consists of a two way switch which connects the motor in star for starting and then in delta for normal running.

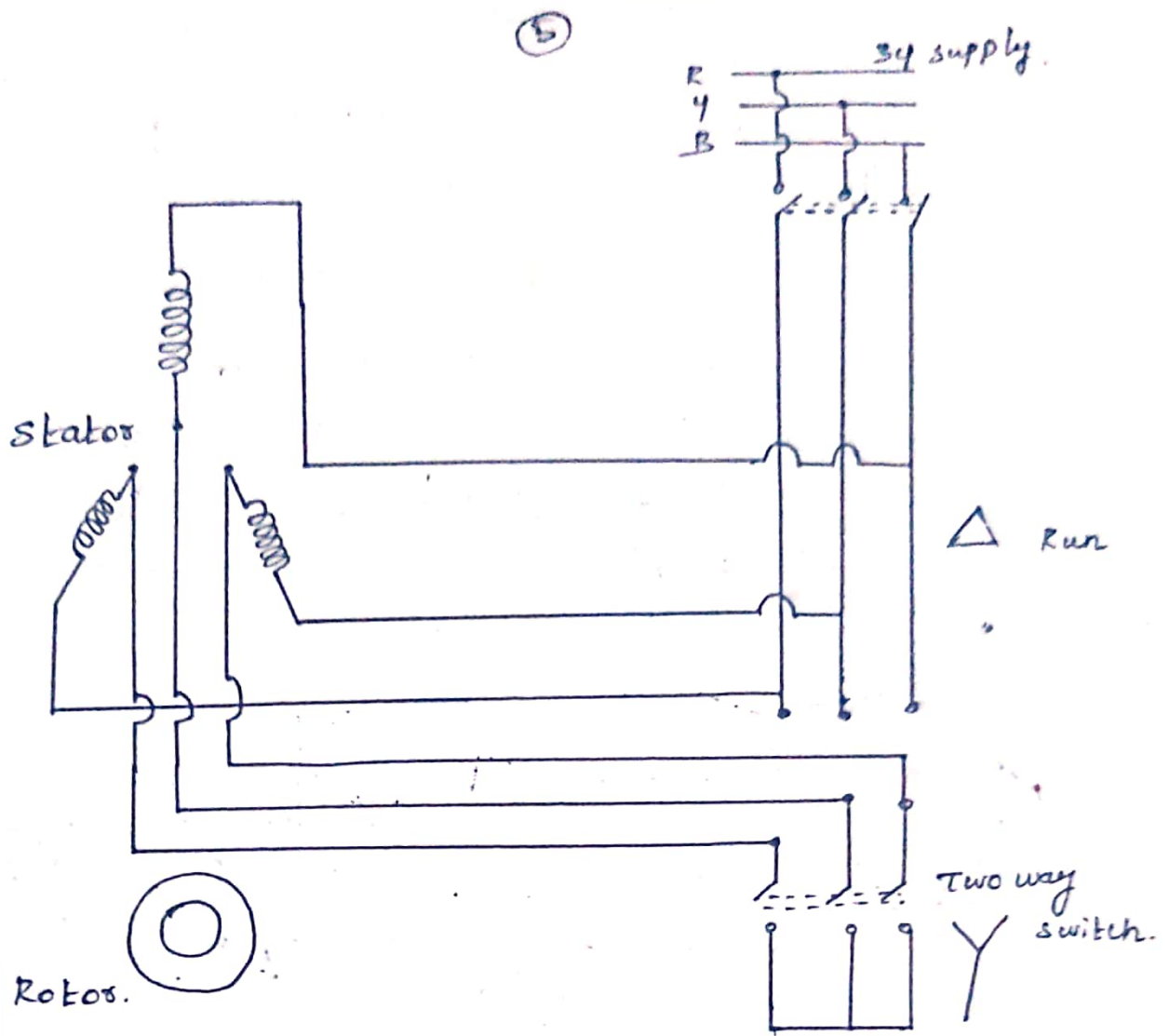
When the two way switch is at START position, the stator windings are connected in star. Therefore the applied voltage is reduced by a factor of  $\frac{1}{\sqrt{3}}$ . Hence starting current is reduced.

Initial starting current and starting torque are given by,

$$\text{Initial starting current, } I_{st} = \frac{1}{\sqrt{3}} I_{sc}$$

$$\begin{aligned} \frac{\text{Starting torque}}{\text{Full load torque}} &= \frac{T_{st}}{T_{fl}} = \left( \frac{I_{st}}{I_{fl}} \right)^2 s_f \\ &= \frac{1}{3} \left( \frac{I_{sc}}{I_{fl}} \right)^2 s_f \end{aligned}$$





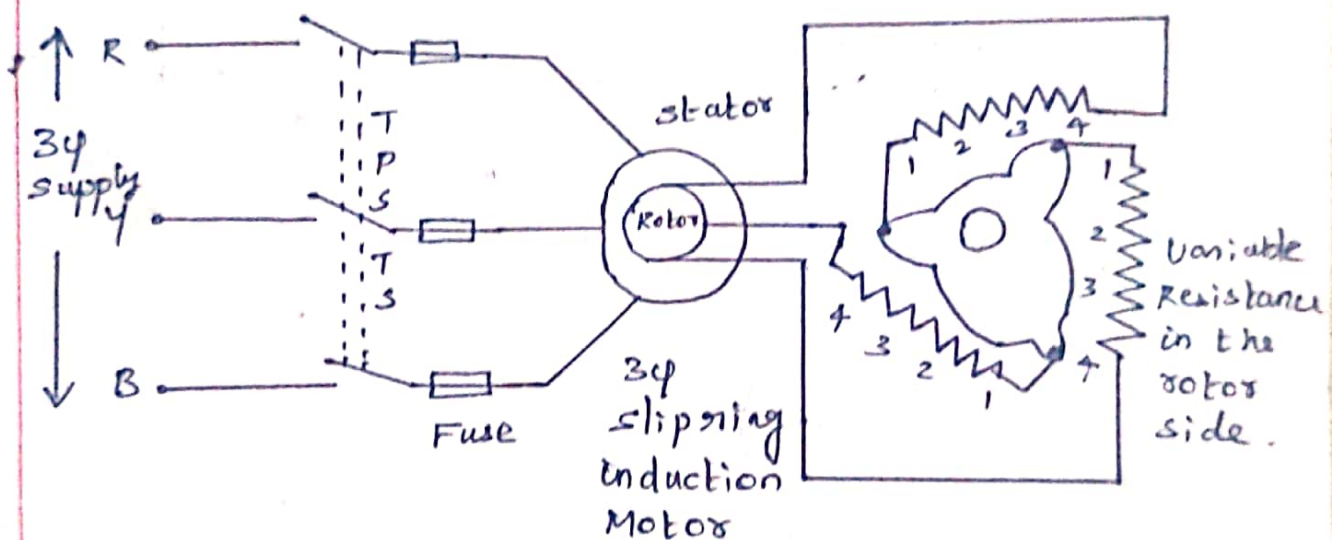
Thus, the motor starting torque is reduced. A star-delta starter is much cheaper compared to auto-transformer starter.

A locking arrangement is also provided so that motor can be started only in star connection.

This starter is mainly used for small and medium size motors.

### Rotor Resistance starter:

This starter can be used only for slip ring induction motor. As shown in fig external or starting resistance is connected in the rotor terminals.



In this method, the motor is always started with full line voltage applied across the stator terminals. The value of starting current is adjusted by introducing a variable resistance in the rotor circuit. At starting, the full resistance is included and hence the starting current is reduced. The resistance is gradually cut out of the rotor circuit as the motor gathers speed.



(6)

## TYPES OF SPEED CONTROL.

Types of stator side control.

1. stator voltage control
2. stator frequency control
3.  $V/f$  control
4. Pole changing method.

Types of rotor side control.

1. Adding external resistance in the rotor circuit.
2. Cascade control.
3. slip power recovery scheme.

## STATOR SIDE CONTROL.

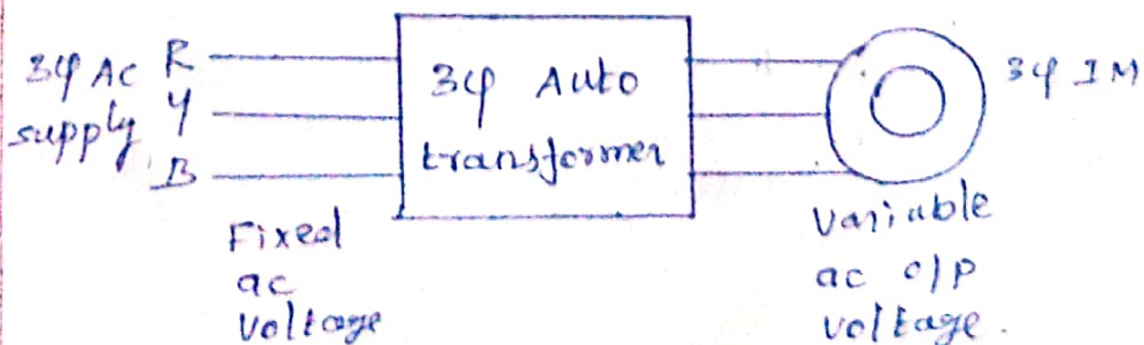
Change in stator voltage.

The speed of the induction motor can be controlled by varying the stator voltage. Here the supply frequency is constant. The stator voltage can be controlled by two methods.

i) Using autotransformer.

ii) Primary resistors connected in series with stator winding.

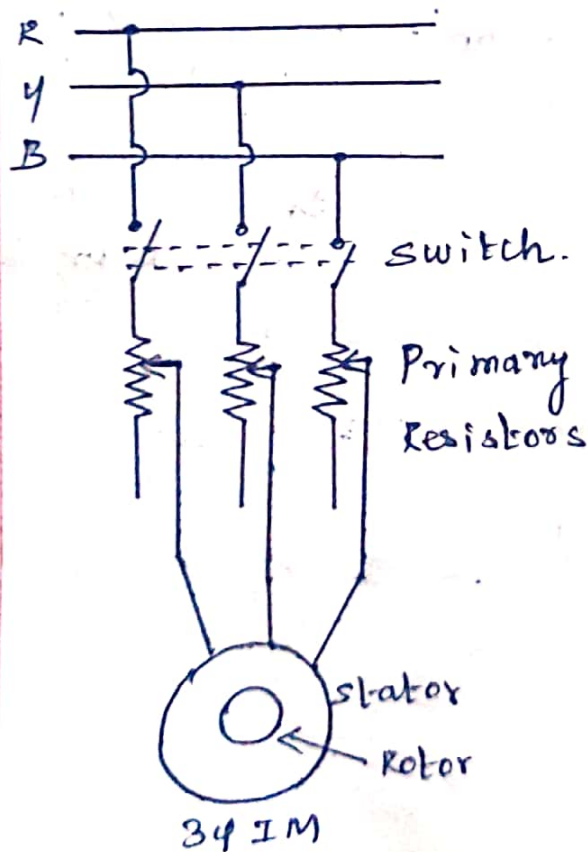
i) Using Auto transformer.



By varying the auto transformer, we can get variable ac o/p voltage without change in supply frequency. The variable voltage is fed to the induction motor. Then the induction motor speed also changes

ii) Primary resistors connected in series with stator winding.

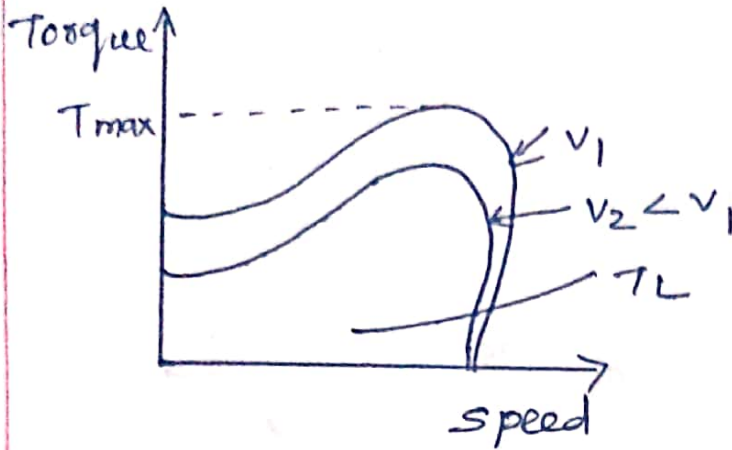
By varying the primary resistance, the voltage drop across the motor terminals is reduced. Then the motor speed can be reduced. The main disadvantage is that more power loss occurs in the primary resistors.



The torque is proportional to the square of its stator voltage i.e.  $T \propto V^2$ . By varying the voltage the torque also changes. This method is not used for wide range of speed control and constant torque load.



(9)

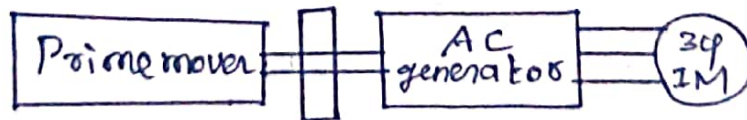


change in stator Frequency :-  
Here, we can vary the input frequency of the motor.

$$\text{Synchronous speed } N_s = \frac{120f}{P}$$

where  $f \rightarrow$  Frequency of the supply voltage  
 $P \rightarrow$  number of poles.

when the supply frequency changes, the motor speed also changes.



The emf  $V$  induced in the stator winding of the induction motor is given by,

$$V = 2\pi f T_1 \phi_{kw}$$

where,  $\phi = \text{Flux/pole}$

$f =$  frequency of stator supply

$kw =$  winding factor.

$T_1 =$  No. of turns in the stator winding.

Here we consider two cases

- i) Low frequency operation at constant voltage.
  - ii) High frequency operation at constant voltage.
- i) Low frequency operation at constant voltage:

By decreasing the supply frequency at constant voltage  $V$  the value of air gap flux increases and the induction motor magnetic circuit gets saturated. Consider the emf equation.

$$V = \text{constant}$$

$$f = \text{decreases}$$

$$\phi = \text{increases}$$

due to this low frequency operation, the following effect take place.

- i) The reactance will be low leading to high motor currents.
  - ii) More losses.
  - iii) very low efficiency.
- ii) High frequency operation at constant voltage:-

with the constant input voltage, if the stator frequency is increased, the motor speed also increases. Due to increase in frequency flux and torque are reduced.



(8)

$V = \text{constant}$

$f = \text{Increases}$

$\phi = \text{Decreases.}$

By increasing the supply frequency of the motor, the following effects will follow.

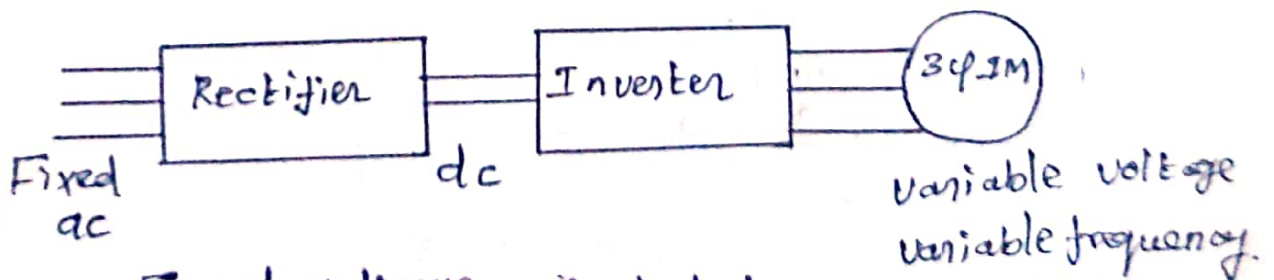
1. The no-load speed increases.
2. The maximum torque decreases.
3. Starting torque reduces.

Voltage / Frequency control:

From the emf equation, the air gap flux is given by  $\phi = \frac{1}{2\pi T_1 k_w} (V/f)$

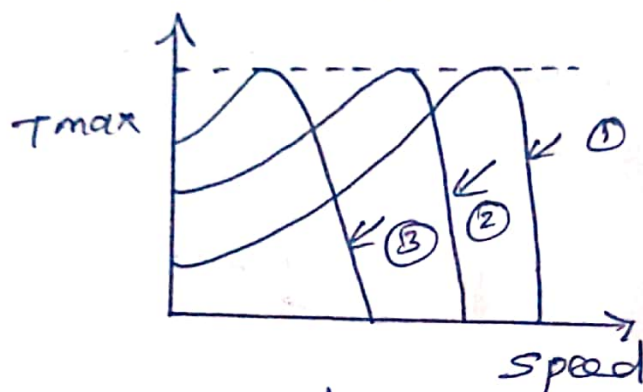
From the above expression, by varying the supply frequency, the air gap flux changes.

To maintain air gap flux constant, the parameters  $V$  and  $f$  must be changed so as to maintain  $(V/f)$  ratio constant. This is known as  $V/f$  control.



Fixed voltage is fed to the rectifier circuit. It converts AC to DC. This DC

Supply is fed to the inverter circuit. It converts DC into variable AC voltage and variable frequency. This o/p is fed to the stator of the induction motor. By varying 'V' and 'f' and maintaining (V/f) ratio constant, the induction motor speed can be changed.



- ① Rated voltage and rated frequency.
- ② reduced voltage and reduced frequency.

This method is applicable only for below base speed.

Changing the number of poles

The synchronous speed of the motor is inversely proportional to the number of poles.

$$N_s \propto \frac{1}{P}$$

By changing the poles, the motor synchronous speed can be varied. Provision for changing the number of poles has to be incorporated at the manufacturing stage and such machines are called pole changing motors or multi speed motors.



(9)

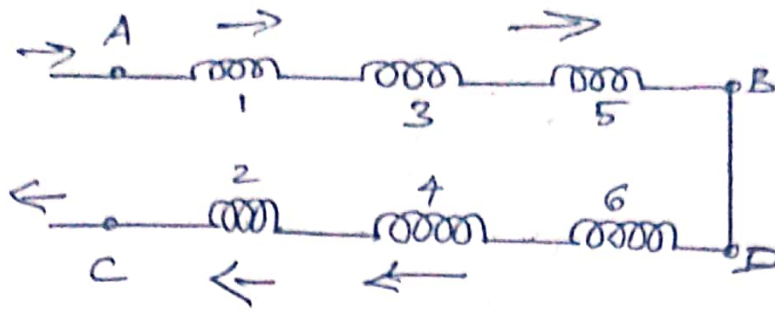


Fig. series connection

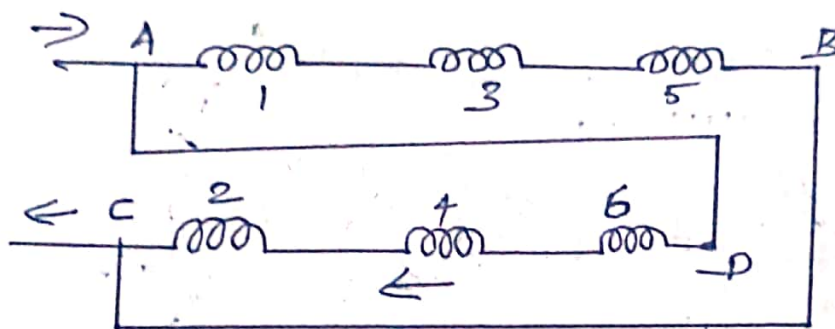


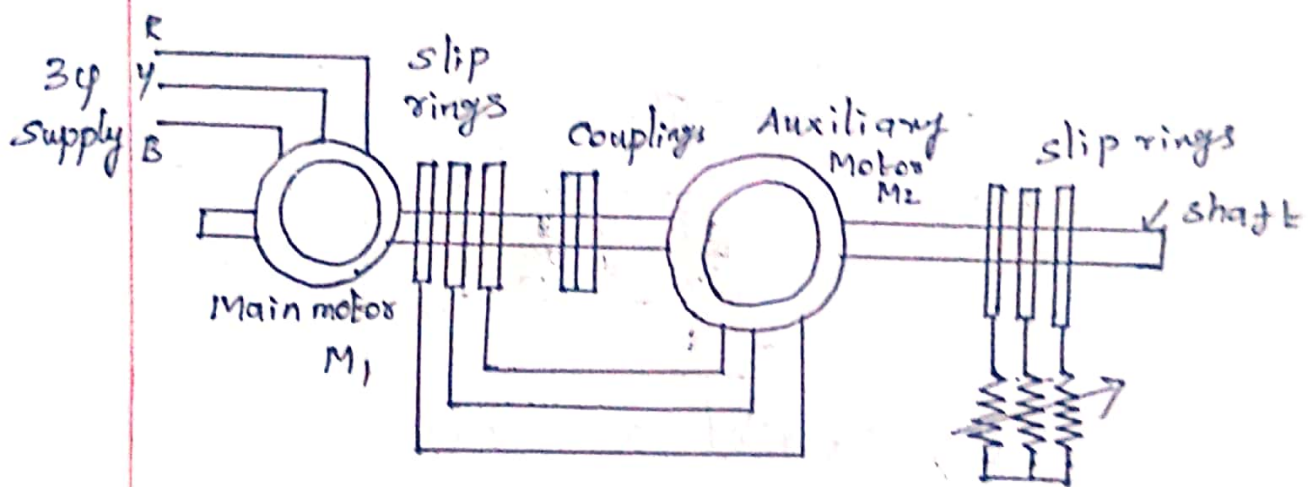
Fig. parallel connection.

Number of poles can be changed by changing these coil groups. Theoretically by dividing the winding into a number of coil groups and bringing out terminals of all these groups, a number of pole numbers can be obtained by rearranging these groups.

## ROTOR SIDE CONTROL

### 1. Cascade control

Another method of speed control of slip ring induction motor is cascade control. It is also known as tandem control.



It consists of two slip ring induction motors. The 1<sup>st</sup> motor is called main motor  $M_1$ . The second motor is called auxiliary motor  $M_2$ .

In these cascading method, if both motors produce the torque in the same direction means, cumulative cascading and opposite direction means differential cascading.

The expression for the speed of the set is derived as follows.



(10)

Let  $P_1$  = number of poles on main motor  $M_1$   
 $P_2$  = number of poles on auxiliary motor  $M_2$   
 $f$  = supply frequency  
 $f_1$  = slip frequency of main motor  $M_1$   
 $f_2$  = slip frequency of auxiliary motor  $M_2$   
 $N$  = speed of both motors.

Synchronous speed of the main motor  $N_{s1}$  is given by,

$$N_{s1} = \frac{120f}{P_1}$$

$N$  = speed of both motors.

Slip for main motor  $M_1$ ,  $s_1 = \frac{N_{s1} - N}{N_{s1}}$

$f_1$  = frequency of rotor induced emf of main motor  $M_1$ .

$$\therefore f_1 = s_1 f$$

The supply frequency of the auxiliary motor  $M_2$  is  $f_1$  i.e.  $f_2 = f_1$ .

$$N_{s2} = \frac{120f_2}{P_2} = \frac{120f_1}{P_2}$$

$$= \frac{120s_1f}{P_2} = \frac{120f}{P_2} \left( \frac{N_{s1} - N}{N_{s1}} \right)$$

Under no load condition

$$N_{s2} = N$$

$$N = \frac{120f}{P_2} \left[ \frac{Ns_1 - N}{Ns_1} \right]$$

$$= \frac{120f}{P_2} \left[ 1 - \frac{N}{Ns_1} \right]$$

$$= \frac{120f}{P_2} \left[ 1 - \frac{N}{\left( \frac{120f}{P_1} \right)} \right]$$

$$N = \frac{120f}{P_2} \left( 1 - \frac{NP_1}{120f} \right) = \frac{120f}{P_2} - \frac{NP_1}{P_2}$$

$$\therefore N \left[ 1 + \frac{P_1}{P_2} \right] = \frac{120f}{P_2}$$

$$\text{(or)} \quad N = \frac{120f}{P_1 + P_2}$$

For cumulative cascade

$$N = \frac{120f}{P_1 - P_2} \quad \text{Differential cascade}$$

Disadvantage

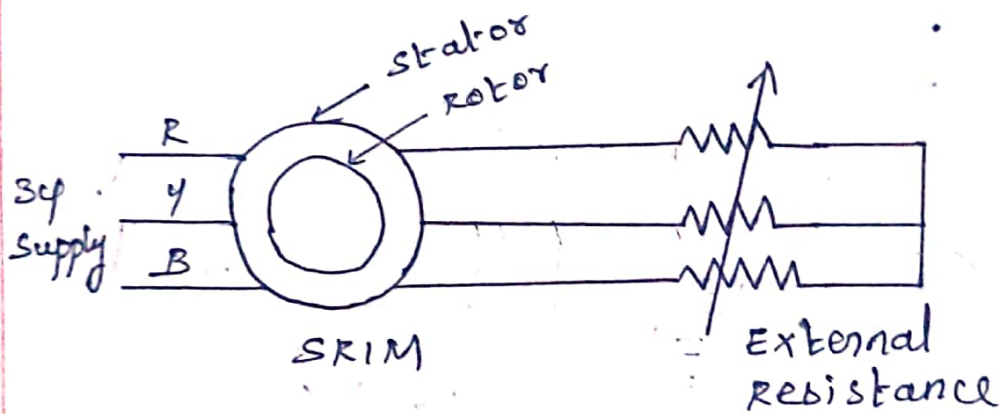
1. Required two motors
2. More expensive
3. wide range of speed control is not possible
4. It cannot be operated when  $P_1 = P_2$  or  $P_1 < P_2$ .



(11)

2. Adding external resistance in the rotor circuit.

This method is applicable only for slip ring induction motor. External resistance can be added in the rotor circuit. Speed control is made by mechanical variation of the rotor circuit resistance.



The torque equation of induction motor is

$$T \propto \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

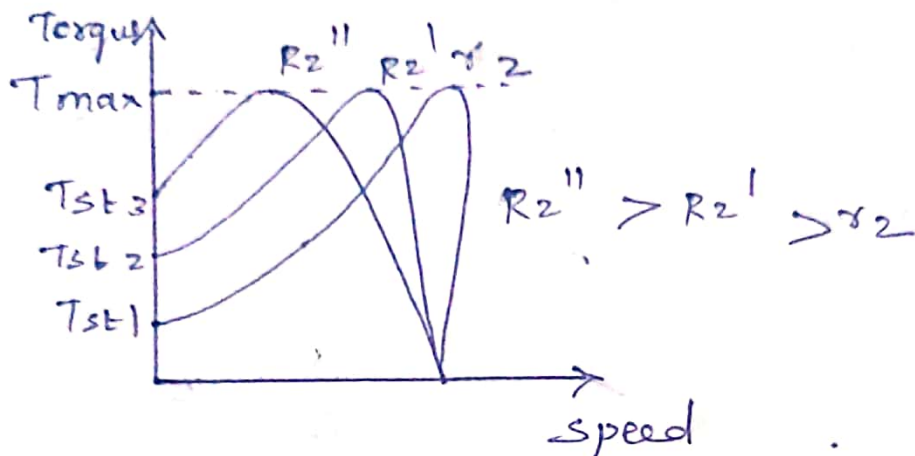
The slip corresponding to maximum torque is given by  $s_m = \frac{R_2}{X_2}$ ,  $s_m \propto R_2$ .

If we add external resistance in the rotor circuit, the slip increases and speed decreases.

The maximum torque equation is

$$T_{max} \propto \frac{E_2^2}{2 X_2}$$

This equation is independent of rotor resistance i.e. by varying the rotor resistance the maximum torque is not affected.



The starting torque of the induction motor is,

$$T_{st} \propto \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Advantages :-

1. Smooth and wide range of speed control.
2. Absence of in-rush starting current.
3. Absence of line current harmonics.
4. High line power factor.
5. Starting torque can be improved.

Disadvantages :-

1. Less efficiency.
2. Unbalance in voltage and current if rotor circuit resistances are not equal.



(12)

### 3. slip power Recovery system

This system is mainly used for speed control of slip ring induction motor.

Rotor air gap power = mechanical power + rotor copper loss

$$P_{ag} = P_m + P_{cu}$$

$$P_{ag} = \omega_s T$$

The air gap flux of the machine is established by the stator supply and it remains practically constant if the stator impedance drop and supply voltage fluctuations are neglected.

The rotor copper loss is proportional to slip. The main drawback of the system is that large amount of slip power is dissipated in the resistance and this reduces efficiency of motor.

This slip power can be recovered and fed back to the supply of an additional motor which is mechanically coupled to the main motor and improves the overall efficiency of the system.

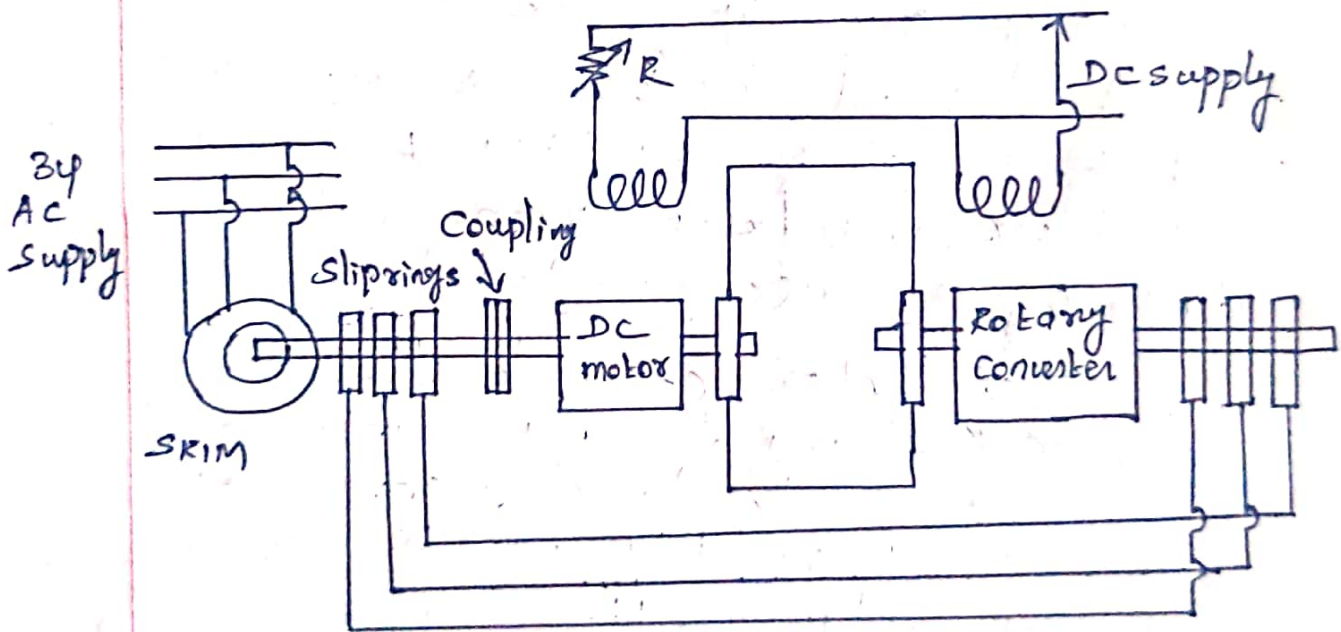
## Types of slip power Recovery system.

1. Kramer system.
2. Scherbius system.

These two systems can further be classified into two methods.

1. conventional method
2. static method.

### a. conventional kramer system.



The system consists of a 3-phase rotary converter and a dc motor. The slip power is converted into dc power by a rotary converter and fed to the armature of a dc motor.

The rotary converter and dc motor are excited from the dc bus bars.



(13)

or from an exciter. The speed of slip ring induction motor is adjusted by adjusting the speed of dc motor with the help of a field regulator.

Advantages :-

1. Any speed, within the working range can be obtained.

2. Better power factor.

b. Static Kramer System.

In this method the slip power is taken from the rotor and it is rectified to dc voltage by 3- $\phi$  diode bridge rectifier. Inductor  $L_d$  smoothes the ripples in the rectified voltage  $V_d$ . This dc power is converted into ac power by using line-commutated inverter.

Advantages :-

1. The static kramer drive has been very popular in large power pump and fan type drives.

2. More economical because the rectifier and inverter have to carry only the slip power of the rotor.

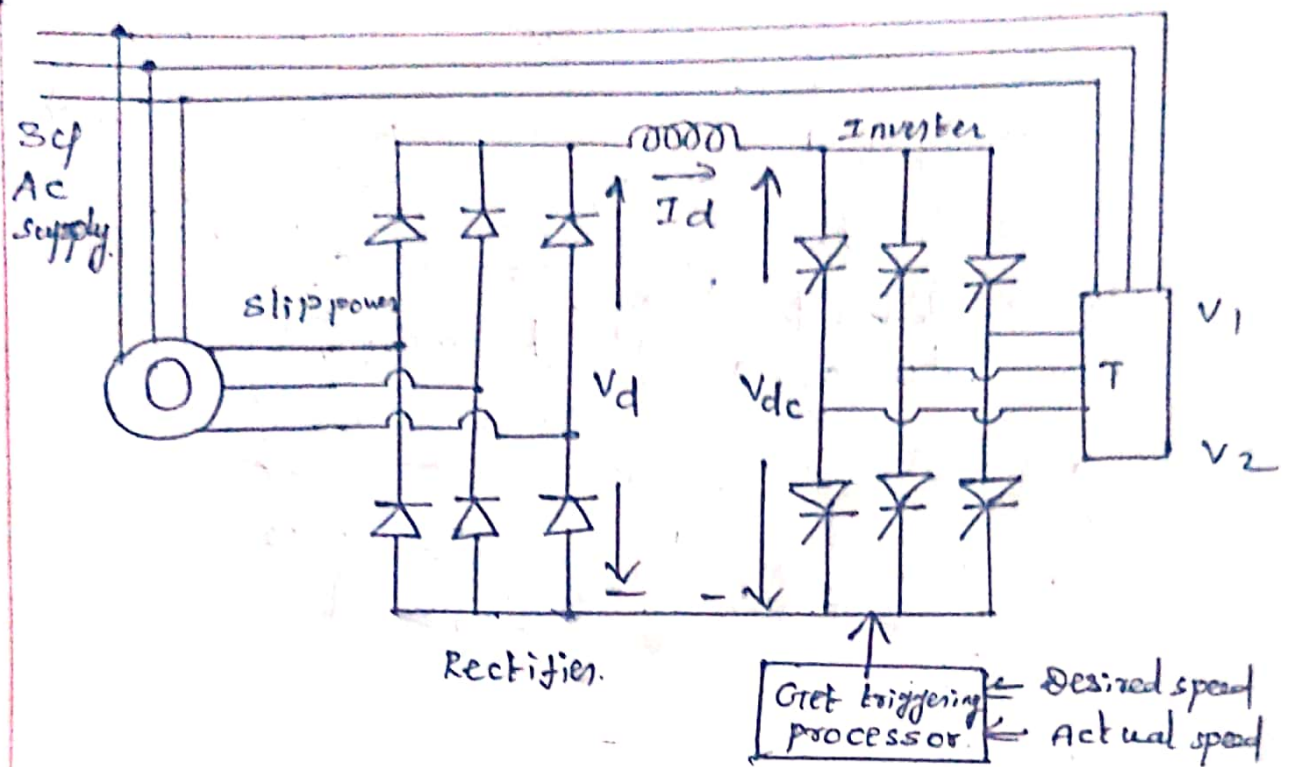


Fig: static kramer system.

### Schenbius System :-

The schenbius system is similar to kramer system but only difference is that in the kramer system the feedback is mechanical and in the schenbius system the return power is electrical.

#### Types :-

- a) conventional schenbius drive
- b) static schenbius drive.

#### a) Conventional schenbius drive :-

This method consists of SCR, rotary converter, dc motor and induction generator. Here, the rotary converter converts slip power into dc power and the dc power fed to the dc motor.



The dc motor is coupled with induction generator. The induction generator converts the mechanical power into electrical power and returns it to the supply line. The SRIM speed can be controlled by varying the field regulator of the dc motor.

3φ  
AC  
Supply

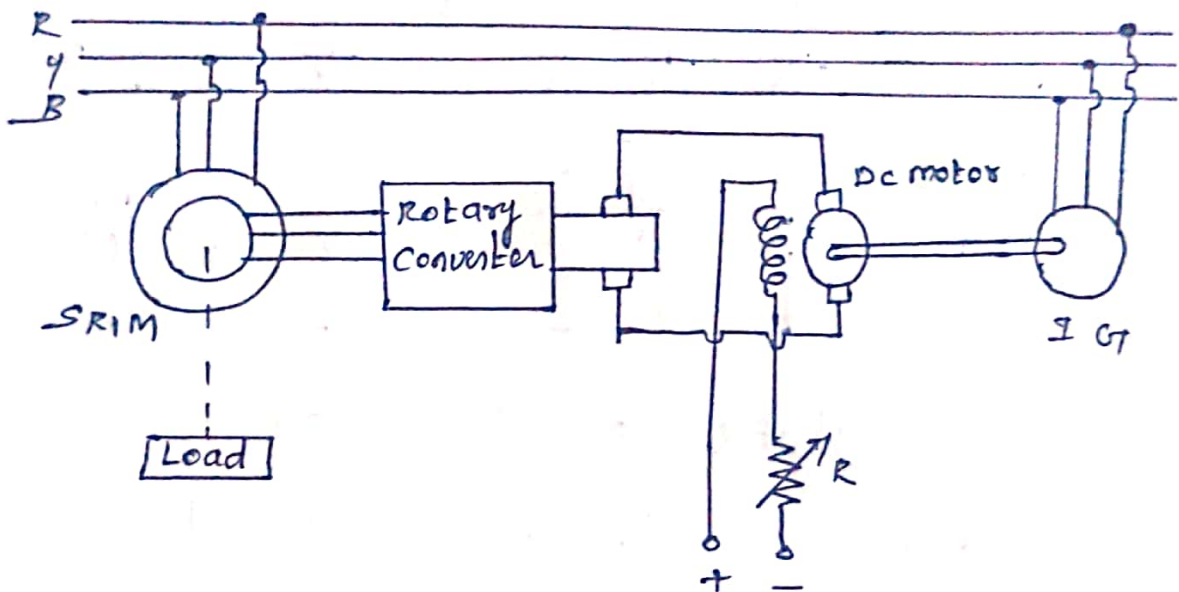


Fig: Conventional Scherbius system.

### Static Scherbius System.

i) sub-synchronous speed operation.

In sub-synchronous speed control of SRIM, slip power is removed from the rotor circuit and is pumped back into the ac supply.

The slip power flows from rotor circuit to bridge 1, bridge 2, transformer and returned to the supply i.e.

slip power  $\rightarrow$  Rectifier  $\rightarrow$  Inverter  $\rightarrow$  Transformer  $\rightarrow$  supply

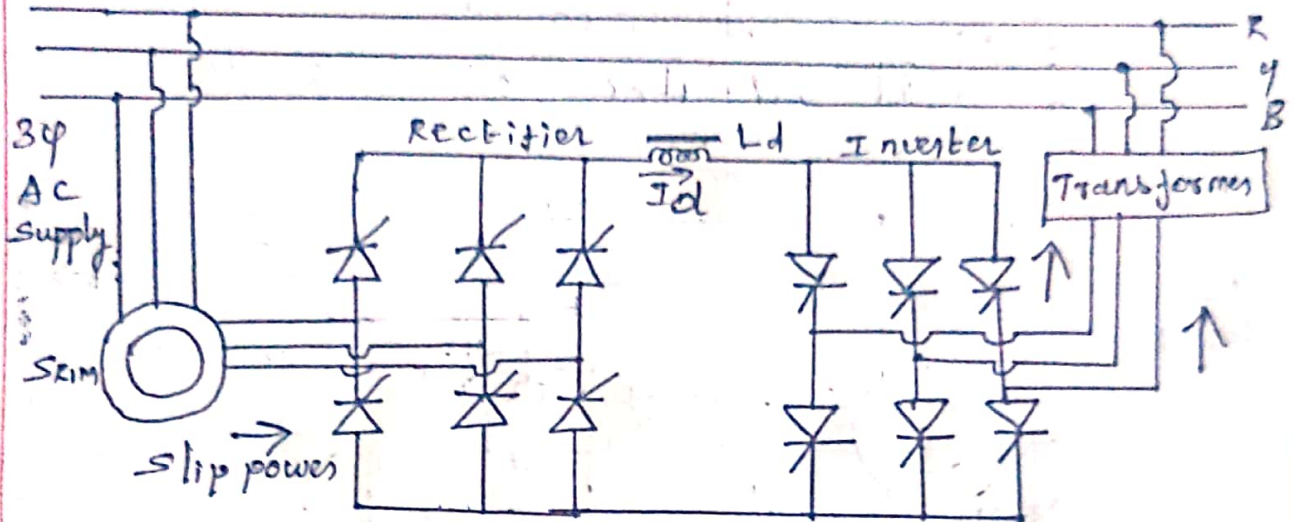


Fig: static scherbius drive

(ii) Super synchronous speed of operation:

In super synchronous speed operation, the additional power is fed into the rotor circuit at slip frequency.

When the machine is operated at super synchronous speed, phase controlled bridge 2 should operate in rectifier mode and bridge 1 in inverter mode. The slip power flows from the supply to transformer, bridge 2, bridge 1 and to the rotor circuit.

Supply  $\rightarrow$  Transformer  $\rightarrow$  Bridge 2  $\rightarrow$  Bridge 1  $\rightarrow$  Rotor circuit



(15)

Example A 4 pole, 50 Hz, 3  $\phi$  slip ring induction motor when fully loaded runs with a slip of 3%. Determine the value of the resistance to be inserted in series per phase in the rotor circuit to reduce the speed by 10% and the new slip. The rotor resistance per phase is 0.2  $\Omega$ .

Solution

$$\text{Synchronous speed } N_s = \frac{120f}{P}$$
$$= \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

$$\text{Speed } N_1 = N_s (1 - s_1)$$
$$= 1500 (1 - 0.03)$$
$$= 1455 \text{ rpm.}$$

$$N_2 = 0.9 \times N_1 = 0.9 \times 1455$$
$$= 1309.5 \text{ rpm.}$$

$$s_2 = \frac{N_s - N_2}{N_s} = \frac{1500 - 1309.5}{1500}$$
$$= 0.127$$

$$\text{Torque } T = \frac{k s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

$$T_1 = \frac{k s_1 E_2^2 R_2}{R_2^2} = \frac{k s_1 E_2^2}{R_2}$$

$$T_2 = \frac{k s_2 E_2^2}{R_2 + r}$$

Given  $T_1 = T_2$

$$\therefore \frac{k s_1 E_2^2}{R_2} = \frac{k s_2 E_2^2}{R_2 + \gamma}$$

$$\therefore \frac{s_1}{R_2} = \frac{s_2}{R_2 + \gamma}$$

$$\frac{0.03}{0.2} = \frac{0.127}{0.2 + \gamma}$$

$$\gamma = 0.646 \Omega$$

Ex: A 3-phase squirrel cage induction motor has a short circuit current equal to 4 times the full load current. Find the starting torque as a percentage of full load torque if the motor is started by (i) Direct switching to the supply mains (ii) A star delta starter (iii) An auto-transformer (iv) A resistance in the stator circuit.

Given:-

$$I_{sc} = 4 I_{fl} \quad s_f = 0.01$$

i) Using DOL starter

$$\frac{T_{st}}{T_{fl}} = \left( \frac{I_{sc}}{I_{fl}} \right)^2 s_f$$

$$T_{st} = \left( \frac{4 I_{fl}}{I_{fl}} \right)^2 \times 0.01 \times T_{fl}$$



$$T_{st} = 16\% \cdot T_{fl}$$

ii) star - delta starter

$$\frac{T_{st}}{T_{fl}} = \frac{1}{3} \left( \frac{I_{sc}}{I_{fl}} \right)^2 s_f$$

$$T_{st} = \frac{1}{3} \left( \frac{4I_{fl}}{I_{fl}} \right)^2 \times 0.01 \times T_{fl}$$

$$T_{st} = 5.33\% \text{ of } T_{fl}$$

iii) Auto-transformer starter.

$$\frac{T_{st}}{T_{fl}} = x^2 \left( \frac{I_{sc}}{I_{fl}} \right)^2 s_f$$

Assume  $x = 1$ ,  $T_{st} = 1^2 \left( \frac{4I_{fl}}{I_{fl}} \right)^2 \times 0.01 \times T_{fl}$

$$= 0.16 T_{fl}$$

$$T_{st} = 16\% \text{ of } T_{fl}$$

iv) resistance in the stator circuit

$$\frac{T_{st}}{T_{fl}} = x^2 \left( \frac{I_{sc}}{I_{fl}} \right)^2 s_f$$

Assume  $x = 1$

$$T_{st} = 1^2 \times \left( \frac{4I_{fl}}{I_{fl}} \right)^2 \times 0.01 \times T_{fl}$$

$$T_{st} = 0.16 T_{fl}$$

$$\therefore T_{st} = 16\% \text{ of full load torque.}$$

UNIT - V



①

## Single phase Induction motor.

Like any other electrical motor asynchronous motor also have two main parts namely rotor and stator.

Stator:-

- stationary parts of Induction motor.
- A single phase AC supply is given to the stator of 1 $\phi$  Induction motor.

Rotor:-

- Rotating part of an induction motor.
- rotor connects the mechanical load through the shaft.
- squirrel cage rotor type.

$$N_s = \frac{120f}{P}$$

where  $f$  = supply frequency  
 $P$  = No. of poles of motor.

Working:-

- Interaction of two fluxes produced the required torque.
- When we apply a 1 $\phi$  AC supply to the stator winding of 1 $\phi$  IM, the alternating current starts flowing through the stator or main winding.

→ This alternating flux is called main flux.

→ According to the Faraday's law of Electro magnetic induction, the main flux links with the rotor conductors and hence cut the rotor conductors.

→ Two fluxes produce the desired torque which is required by the motor to rotate.

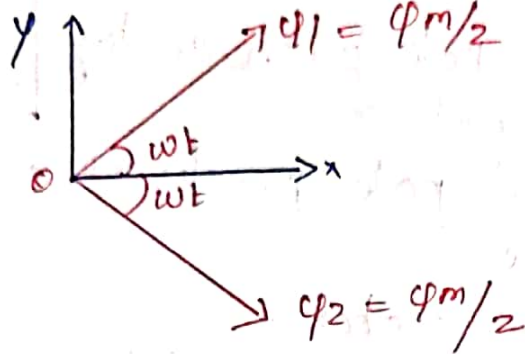
→ According to double revolving field theory, we can resolve any alternating quantity in to two components. Each component has a magnitude equal to the half of the maximum magnitude of the alternating quantity, and both these components rotate in the opposite direction to each other.

→  $\phi$  can be resolved in to two components  $\frac{\phi_m}{2}$  and  $-\frac{\phi_m}{2}$ .

$$\phi_r = \phi_f + \phi_b$$



Explain the double field revolving theory for operation of 1 $\phi$  induction motor.  
 NOV-DEC 2012  
 → Represented by two revolving fluxes.



APRIL - MAY 2015  
 NOV-DEC 2015

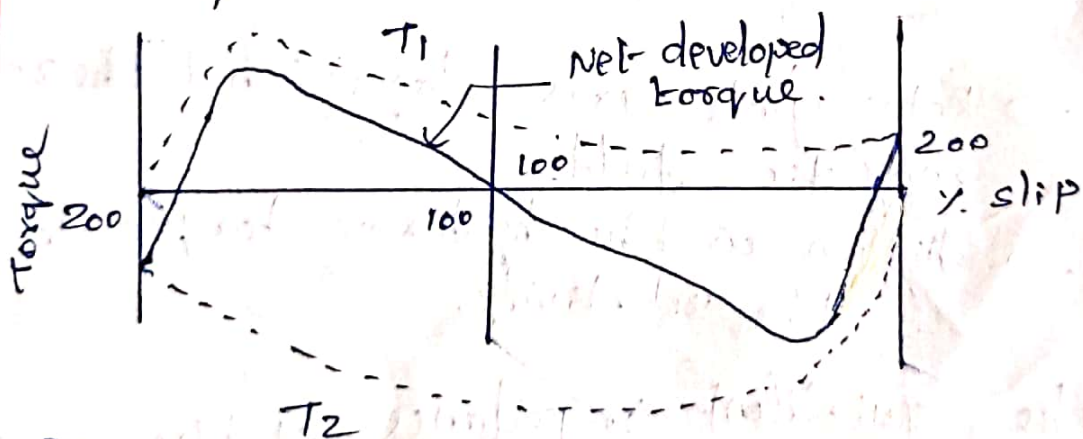
X Component =  $\phi_m \cos \omega t$

Y Component = 0

Resultant flux =  $\sqrt{(\phi_m \cos \omega t)^2 + 0^2} = \phi_m \cos \omega t$

(i) Rotor at standstill

→ Rotor standstill, stator connected with 1 $\phi$  AC supply.



ii) Rotor running

→ spinning the rotor by auxiliary circuit

Forward slip  $s_f = \frac{N_s - N}{N_s} = s$

backward slip  $s_b = 2 - s$

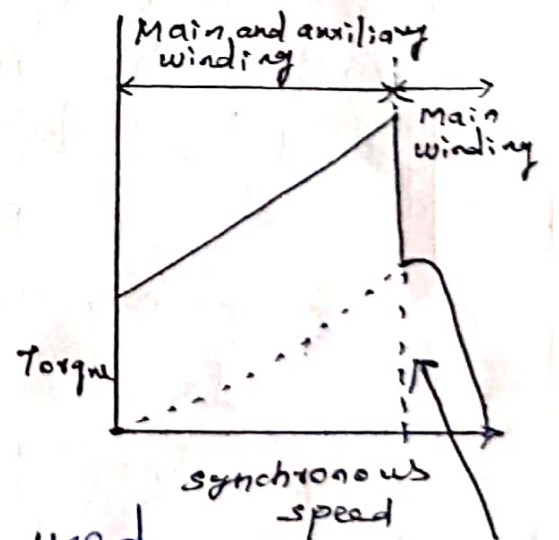
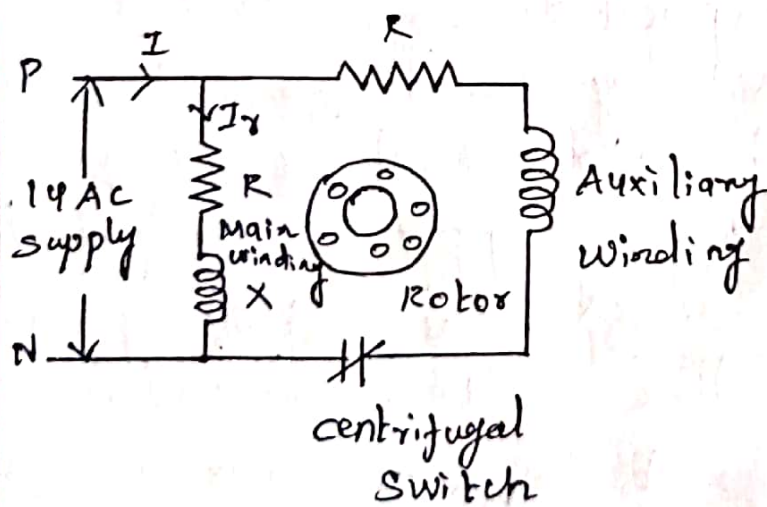
(3)

## Types

1. split phase induction motor
2. Capacitor start induction motor
3. Capacitor start capacitor run IM
4. Shaded pole IM
5. Permanent split capacitor motor

1. split phase induction motor.

It consists of two stator windings. One is the main winding or running winding and another is auxiliary winding or starting winding. These two windings axes are displaced by  $90^\circ$  electrical degrees.



The auxiliary winding is used only for starting period.

centrifugal operates.



When the motor speed is about 75% of synchronous speed, the auxiliary winding is disconnected from the circuit. This is done by connecting a centrifugal switch in the auxiliary circuit. After this motor runs because of main winding only.

Application:-

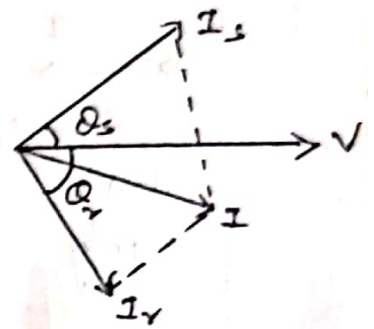
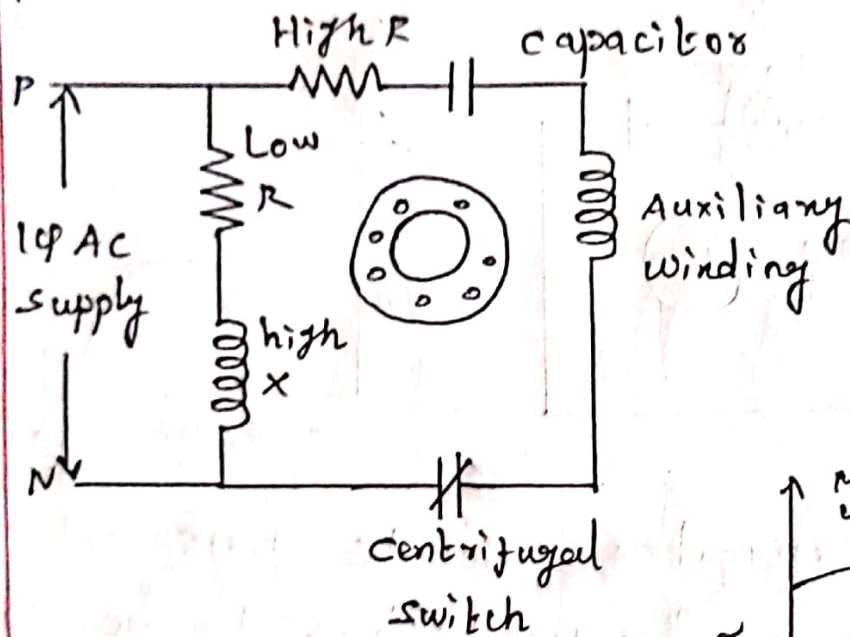
- (i) fans
- (ii) blowers
- (iii) Centrifugal pumps
- (iv) Washing machines.

2 Capacitor start single phase Induction motor.

Here, a capacitor is connected in series with the auxiliary windings. It is also used to get higher starting torque.

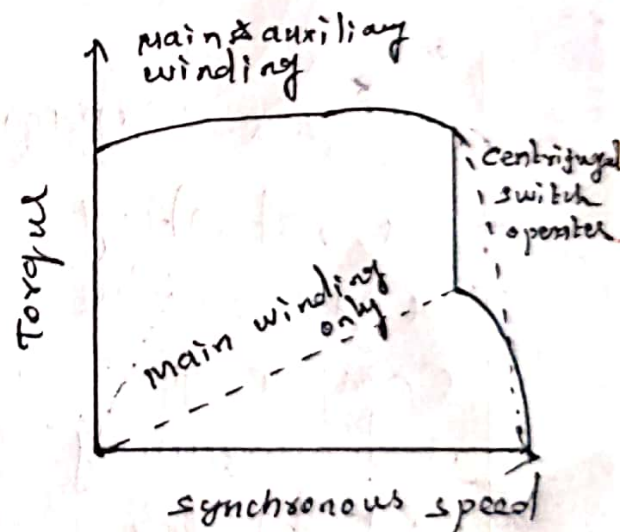
single phase supply is applied to the two windings. The starting current  $I_s$  leads the line voltage, because of the capacitor present in the auxiliary winding. The running current  $I_r$  lags the line voltage.

⑨  
 The phase displacement between the two currents is approximately equal to  $90^\circ$  during starting. The direction of rotation of the motor can be changed by changing the connections of one of the windings.



### Applications :-

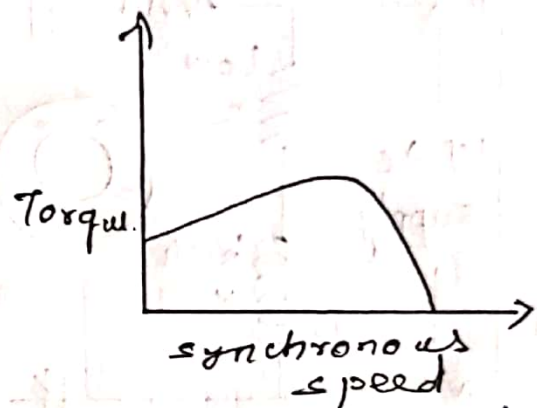
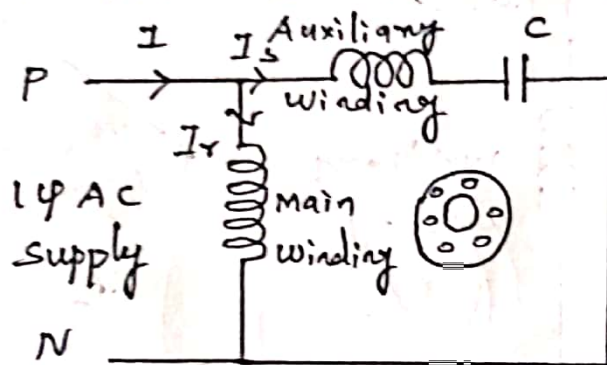
1. Compressors
2. Pumps
3. Conveyors
4. Refrigerators
5. Air conditioning equipments
6. Washing machines.





### 3. Capacitor run single phase Induction motor.

In this motor, a capacitor is permanently connected in series with the auxiliary winding. Here, the centrifugal switch is not needed and therefore the cost of the motor is less.



The capacitor is AC paper oil type. The starting torque has to be sacrificed because the capacitor chosen is a compromise between the best starting and running conditions.

#### Advantages

1. High power factor at full load.
2. High full load efficiency.
3. Increased pull out torque
4. Low full load line current

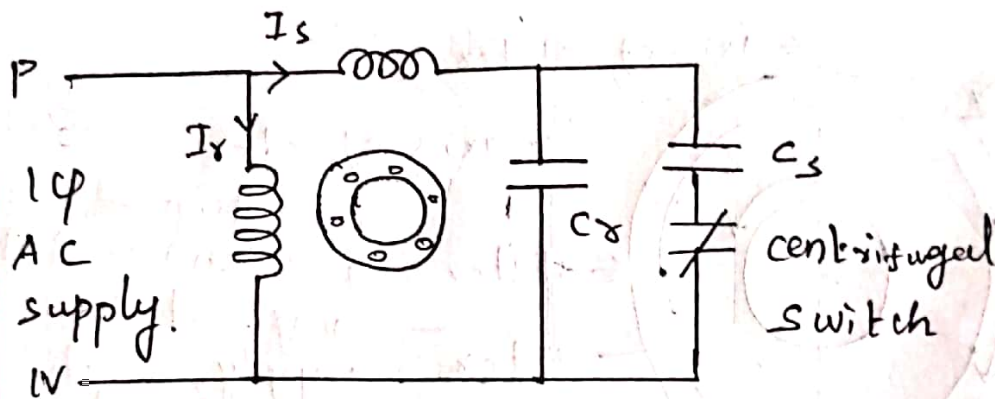
#### Applications :-

1. Fans
2. Blowers
3. Centrifugal pumps.

(5)

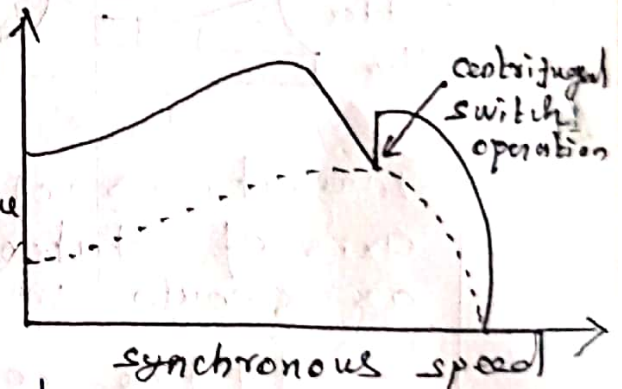
4. Capacitor start capacitor run motor.

Here, two capacitors are used. One capacitor  $C_s$  is used for starting purpose and another capacitor  $C_r$  is used for running purpose. In this motor, we can get high starting torque because of two capacitors.



$C_s$  - large value  
 $C_r$  - small

The capacitor  $C_s$  is used for developing high starting torque and capacitor  $C_r$  is used to improve the power factor.



Advantages :-

1. High starting torque
2. High efficiency
3. High power factor.

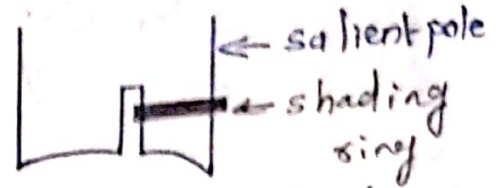
Application :-

1. compressors
2. Pumps
3. conveyors
4. Refrigerators.



(5) (i) shaded pole induction motor <sup>NOV-DEC 2014</sup>  
 APRIL - MAY 2015  
 → sweeping of the flux across the pole face.

→ salient pole with shading band.

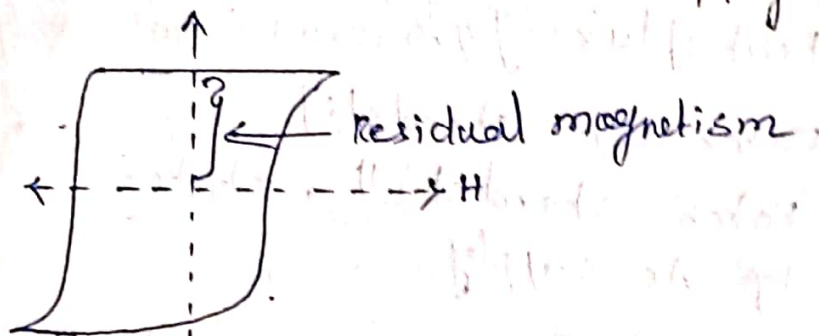


→ shaded pole produces a weak starting torque.

(ii) Hysteresis motor.

→ Rotor made up of hard magnetic material  
 → Rotor having high hysteresis loop area.

May-June 2016



→ Rotor pole axis lags behind the rotating magnetic field.

→ Higher is the hysteresis torque if higher is the retentivity.

3. The equivalent impedance of the main and auxiliary winding in a capacitor motor are  $(15 + j22.5) \Omega$  and  $(50 + j120) \Omega$  respectively while capacitance of the capacitor is  $12 \mu F$ . Determine the line current at starting on a

(7)

May-June 2013

230V, 50Hz Supply.

Soln:-

$$I_m = \frac{V}{Z_m} = (4.718 - j7.077) \text{ A}$$

$$X_c = \frac{1}{2\pi f C} = 265.258 \Omega$$

$$Z_{ac} = Z - jX_c = 50 - j145.258 \Omega$$

$$I_a = \frac{V}{Z_{ac}} = \frac{230}{50 - j145.258}$$

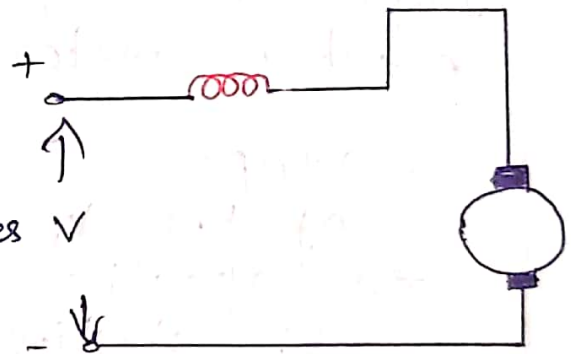
$$= (0.487 + j1.416) \text{ A}$$

$$I_L = I_m + I_a = 7.69 \angle -47.4^\circ \text{ A}$$



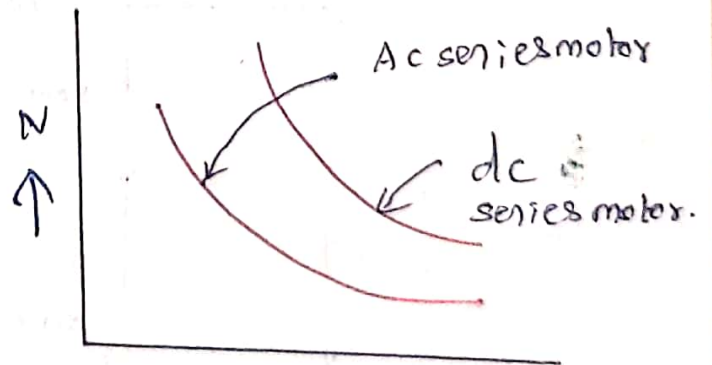
(ii) AC series motor.

- Ability to run on both direct and alternating current.
- Field flux, armature current reverse simultaneously.
- power factor of the motor will be less.
- sparking at the brushes will occur.



### Applications

- \* sewing machine
- \* vacuum cleaners
- \* electric locomotives.



Torque speed characteristics.

(8)

## STEPPER MOTOR.

A stepper motor is a brushless DC motor whose rotor rotates through a fixed angular step in response to each input current pulse received by its controller.

A stepper motor is a digital actuator, whose input is in the form of programmed energization of the stator windings and whose output is in the form of discrete angular rotation.

Torque rating  $\rightarrow$  1 mN-m to 40 N-m  
output ranges  $\rightarrow$  1 W - 2500 W.

Stepper motor consists of two parts i.e. stator and rotor. Stator has phase windings. Rotor has no electrical windings, commutator or brushes.

Step angle :-

Step angle is defined as the angle through which the stepper motor shaft rotates for each command pulse.



It is denoted as  $\beta$ .

Minimum step angle  $0.72^\circ$

Maximum step angle  $90^\circ$

$$\textcircled{i} \quad \beta = \frac{N_s - N_r}{N_s N_r} \times 360^\circ$$

where,  $N_s$  = Number of stator poles or stator teeth.

$N_r$  = Number of rotor poles or rotor teeth.

$$\textcircled{ii} \quad \text{step angle } \beta = \frac{360^\circ}{m N_r}$$

where  $m$  = Number of stator phases.

Resolution :-

It is defined as the number of steps needed to complete one revolution of the rotor shaft.

$$\therefore \text{Resolution} = \frac{\text{Number of steps}}{\text{Revolution}} = \frac{360^\circ}{\beta}$$

Stepping rate :-

→ The number of steps per second

Classification of stepper motor.

- (i) variable reluctance stepper motor
- (ii) Permanent Magnet stepper motor.
- (iii) Hybrid stepper motor.

variable reluctance stepper motor. (VR)

The VR stepper motor consists of stator and rotor. The stator windings are wound on the stator poles. The rotor carries no windings. Rotor poles are of a ferromagnetic material. The rotor is a salient pole type.

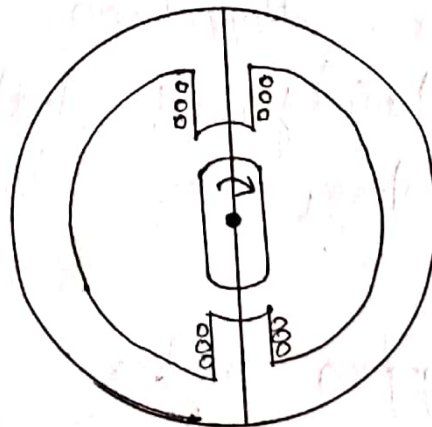


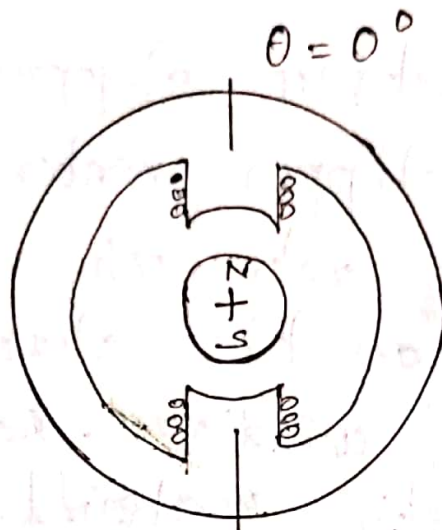
Fig: VR motor.

This stepper motor is called variable reluctance motor because the reluctance of the magnetic circuit formed by the rotor and stator both varies with angular position of the rotor.



Permanent Magnet stepper motor.

In this type of motor, rotor is a cylindrical type. stator windings are wound on the stator poles.



PM motor.

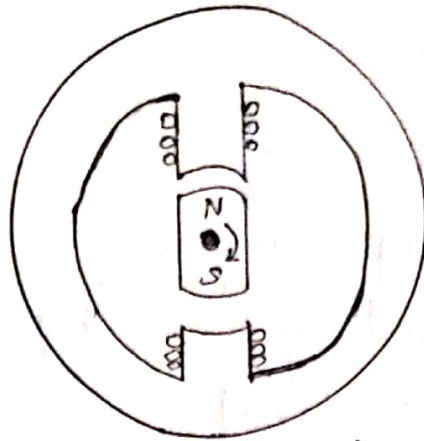
The motor direction of rotation depends on the polarity of stator current. Its main advantage is its low manufacturing cost.

Hybrid stepper motor.

This is the most popular type of stepper motor. Here, the rotor is permanent magnet. It is a salient pole type rotor.

$$\theta = 0^\circ$$

(10)



Hybrid motor.

The hybrid stepper motor combines the features of VR stepper motor as well as PM stepper motor.

Applications :-

1. Instrumentation applications
2. Numerical control of machine tools.
3. Electro medical
4. Computer peripherals.

Advantages :-

1. No accumulative position error.
2. It is mechanically simple
3. Free from contamination
4. Relatively rugged and durable.
5. Maintenance free.



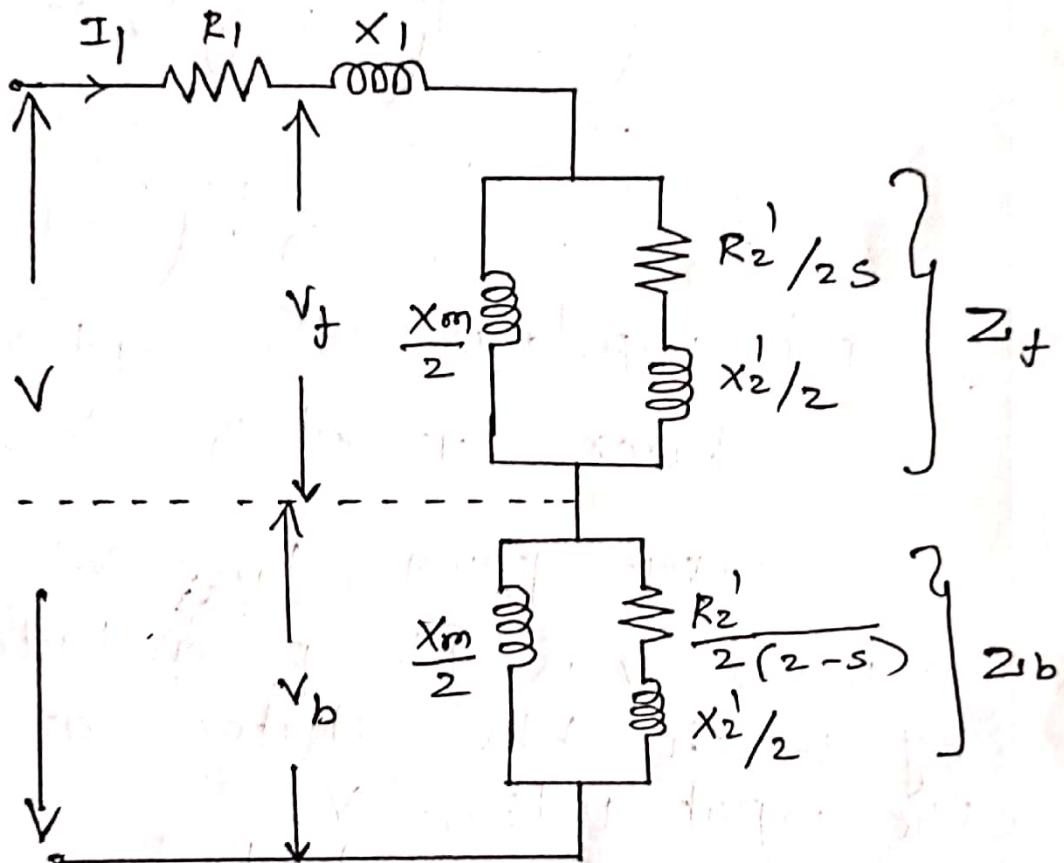
## Disadvantages:-

1. Low efficiency
2. Fixed step angle.
3. No flexibility in step resolution.
4. High overshoot.
5. Limited power output and size available.

## Equivalent circuit :-

The equivalent circuit of a single phase induction motor can be developed by using double revolving field theory.

By using the equivalent circuit, the performance of the single phase induction motor can be obtained.



By using this equivalent circuit, we can calculate the stator current, input power, developed power, developed torque and efficiency for a particular speed.



$$\text{Let } Z_f = R_f + jX_f$$

$$= \frac{\left( j \frac{X_m}{2} \right) \left( \frac{R_2'}{2s} + j \frac{X_2'}{2} \right)}{\frac{R_2'}{2s} + j \left( \frac{X_2'}{2} + \frac{X_m}{2} \right)}$$

$$Z_b = R_b + jX_b$$

$$= \frac{\left( j \frac{X_m}{2} \right) \left( \frac{R_2'}{2(2-s)} + j \frac{X_2'}{2} \right)}{\frac{R_2'}{2(2-s)} + j \left( \frac{X_2'}{2} + \frac{X_m}{2} \right)}$$

$$\text{input impedance } Z_{in} = Z_1 + Z_f + Z_b$$

$$\text{stator current } I_1 = \frac{V}{Z_{in}}$$

$$\text{input power } P_{in} = VI_1 \cos \phi$$

where  $\phi$  is the power factor angle by which the stator current  $I_1$  lags the input voltage  $V$ .

Air gap power due to forward field

$$P_{agf} = I_1^2 R_f$$

Air gap power due to backward field

$$P_{agb} = I_1^2 R_b$$

$$\text{Forward torque } T_f = \frac{P_{agf}}{\omega_s}$$

(12)

Resultant torque  $T = T_f - T_b$

$$= \frac{P_{agf}}{\omega_s} - \frac{P_{agb}}{\omega_s}$$

Mechanical power developed  $P_m = T \omega_m$

$$= T \omega_s (1-s)$$

$$= (P_{agf} - P_{agb})(1-s)$$

shaft power o/p =  $P_{out} = P_m - P_{md}$

where  $P_{md}$  is the mechanical loss.

Efficiency  $\eta = \frac{P_{out}}{P_{in}}$

Rotor copper loss due to forward flux  
 $= s P_{agf}$

Rotor copper loss due to backward flux  
 $= (2-s) P_{agb}$

$\therefore$  Total rotor copper loss  $P_{cu} = s P_{agf} + (2-s) P_{agb}$

The total air gap power is the sum of the air gap powers absorbed from the stator by the two air gap fields,

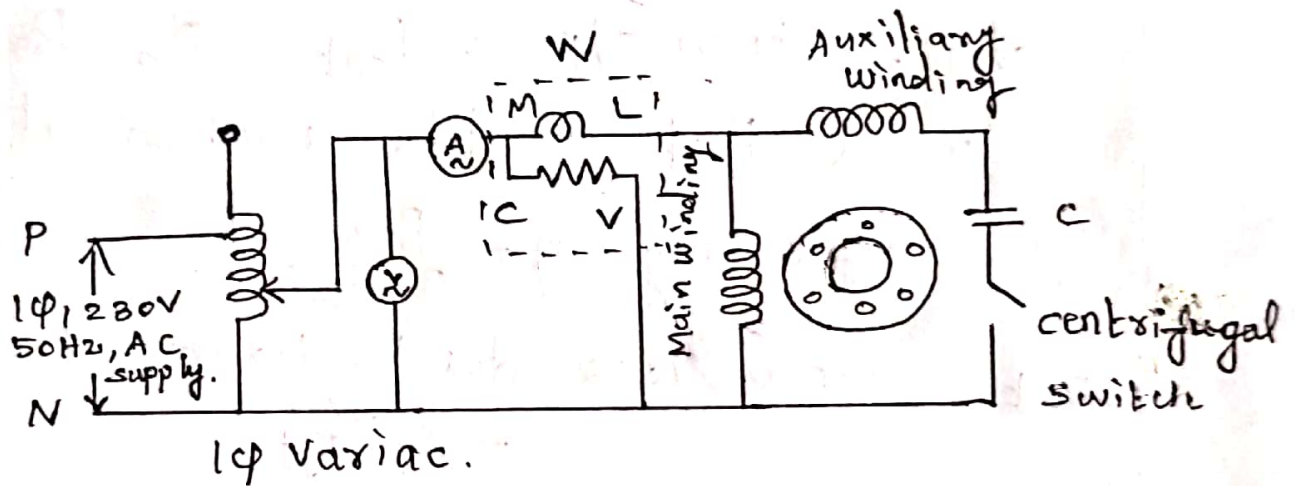
$$P_{ag} = P_{agf} + P_{agb}$$

$$= P_{in} - I_1^2 R_1$$

## NO Load and Blocked Rotor Test:-

These two tests are mainly used to determine the parameters of the equivalent circuit of single phase induction motor.

The resistance of the main winding  $R_1$  can be measured by voltmeter-ammeter method at full load current.



## Blocked Rotor Test :-

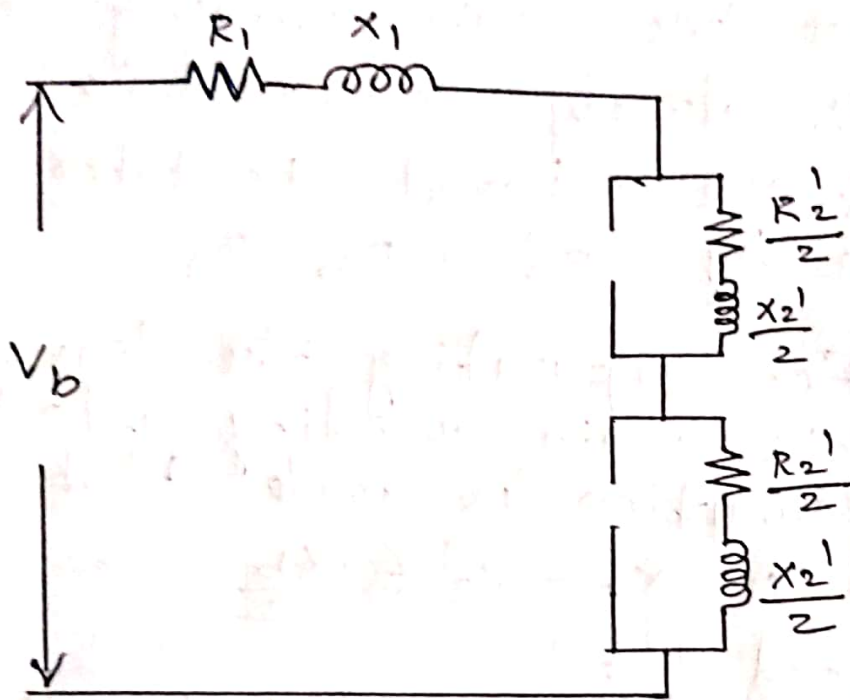
In blocked rotor test, the rotor should be blocked. Here, we have to apply reduced voltage to the main winding, so that rated current flows through the main winding. Normally auxiliary winding is kept open during this test.

Here, the applied voltage is very low and the slip at standstill is unity.



(13)

Therefore the excitation branch can be neglected from the equivalent circuit.



Under this test,

$V_b$  = Reduced voltage applied to motor under blocked rotor condition.

$I_b$  = Full load current

$W_b$  = Full load power.

Impedance under blocked rotor condition,

$$Z_b = \frac{V_b}{I_b}$$

Resistance under blocked rotor condition

$$R_b = \frac{W_b}{I_b^2}$$

Reactance under blocked rotor condition

$$X_b = \sqrt{Z_b^2 - R_b^2}$$

$$\text{Also } R_b = R_1 + R_2'$$

$$X_b = X_1 + X_2'$$

Since, the main winding resistance  $R_1$  is already measured, the rotor resistance referred to stator  $R_2'$  is given by  $R_2' = R_b - R_1$

For separating the leakage reactance of the main winding and rotor winding, an assumption is made that they are equal i.e.  $X_1 = X_2' = \frac{X_b}{2}$

No Load Test:-

Under no load test, the 1 $\phi$  IM rotates at rated voltage and rated frequency. Under this condition, the slip is very small. Now,

$V_0$  = Rated voltage applied to the motor.

$I_0$  = No load current

$W_0$  = No load power.

Impedance under no load condition

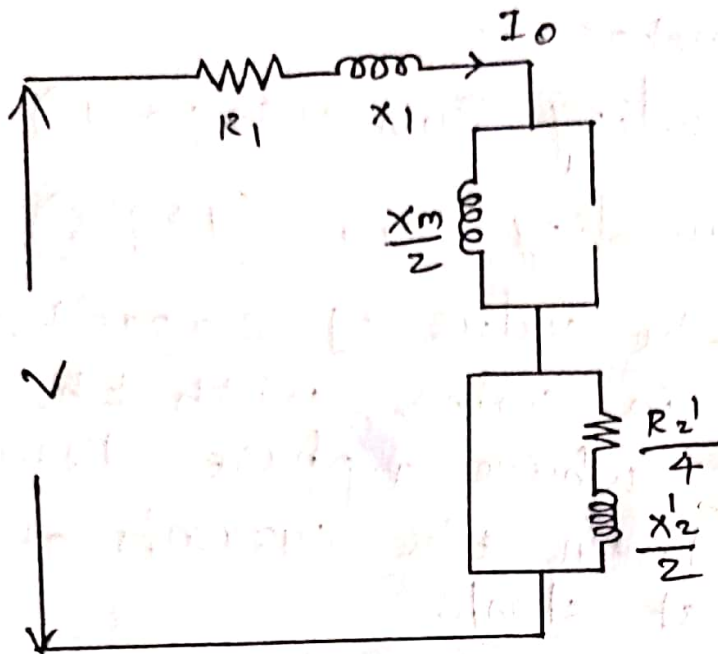
$$Z_0 = \frac{V_0}{I_0}$$

Resistance under no load condition

$$R_0 = \frac{W_0}{I_0^2}$$

Reactance under no load condition

$$X_0 = \sqrt{2I_0^2 - R_0^2}$$



From Fig. we have

$$X_0 = X_1 + \frac{X_m}{2} + \frac{X_2'}{2}$$

From blocked rotor test,

$$X_1 = X_2' = X_b/2$$

on substituting the values of  $X_1$  and  $X_2'$  in  $X_0$ ,

$$X_0 = \frac{X_b}{2} + \frac{X_m}{2} + \frac{X_b}{4}$$

$$X_m = 2X_0 - \frac{3}{2} X_b$$

The rotational losses or mechanical losses

$$P_{mL} = W_0 - I_0^2 \left( R_1 + \frac{R_2'}{4} \right)$$

By using these two tests, we can calculate torque developed, input power, output power, power factor and efficiency.



Ex: The main and auxiliary winding impedance of a 50 Hz, capacitor start single phase induction motor are,

$$\text{Main winding } Z_{1m} = (3 + j2.7) \Omega$$

$$\text{Auxiliary winding } Z_{1a} = (7 + j3) \Omega$$

Determine the value of capacitor to be connected in series with the auxiliary winding to achieve a phase difference of  $\alpha = 90^\circ$  between the currents of the two windings at start.

Solution

$$Z_{1a} = 7 + j3 + jX_c = 7 + jX$$

$$X \rightarrow \text{net reactance. } Z_{1m} = 3 + j2.7 \\ = 4.03 \angle 42^\circ \Omega$$

$I_m$  lags behind  $v$  by  $42^\circ$ .

$$90 - 42 = 48^\circ$$

for auxiliary winding

$$\tan \phi_{1a} = \frac{X}{R} = \tan 48^\circ = \frac{X}{7}$$

$$X = 7 \times 1.11 = 7.77 \Omega \text{ (capacitive)}$$

$$X_c = 7.77 + 2 = 10.77 \Omega$$

$$\therefore X_c = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_c} = \frac{1}{2\pi \times 50 \times 10.77}$$

$$C = 295.55 \mu\text{F}$$

## Repulsion Motor: - (15)

Repulsion motors are similar to series motors except that the rotor and the stator windings are inductively coupled i.e., the rotor current is obtained by transformer action from stator.

Types: -

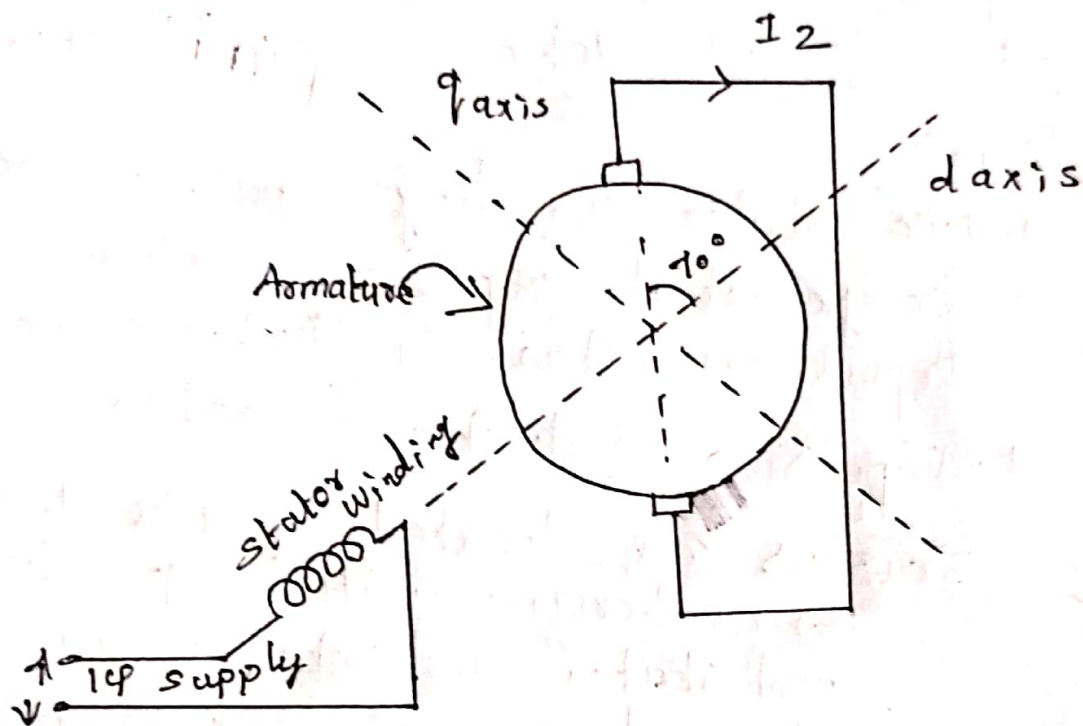
1. Two stator winding repulsion motor.
2. Compensated repulsion motor.
3. Repulsion start induction motor.
4. Repulsion induction motor.

Brushes are located in the d-axis  
→ no torque will be produced and stator and rotor fields are aligned. EMF is maximum.

Brushes are located in the q-axis  
→ The emf induced in the armature winding would add up to zero and current  $I_2$  is zero and hence no torque will be produced.

As it is essential that both an armature current should flow and an angular displacement must exist between two fields the brushes are located in an intermediate position making a large angle of about  $70^\circ$

with the d-axis. This appears like repulsion between the stator and rotor fields. That is why it is called repulsion motor.



Dis advantage

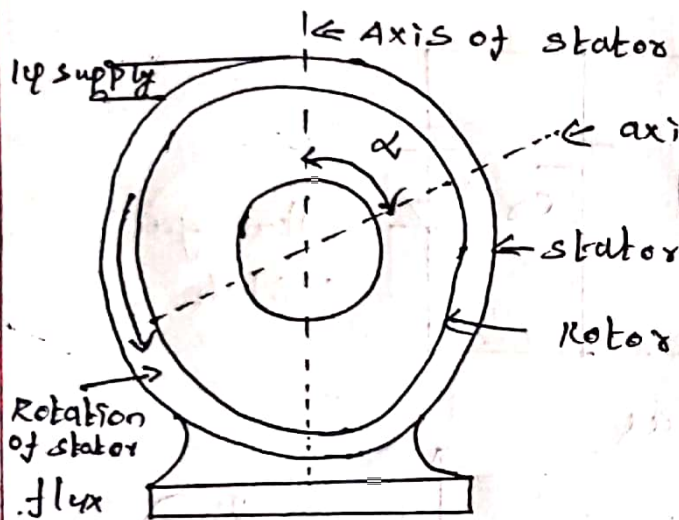
- i) Speed variations with the variations in load dangerously high at no load.



## HYSTERESIS MOTOR.

It is a single phase motor whose operation depends upon the hysteresis effect and on the presence of continuously revolving magnetic flux.

A rotor consisting of a smooth cylinder of magnetically hard steel, without winding or teeth



Operation:-

1. When the stator is energised from AC supply, revolving speed is produced.
2. Due to hysteresis effect, the axis of

magnetises of rotor will lag behind the axis of stator field by hysteresis lag angle as shown in figure!

If the rotor is stationary, the starting torque produced is given by,

$$T_s \propto \phi_s \phi_r \sin \alpha$$

3. After reach synchronism, the motor continues to run at synchronous speed.

Application:

1. Electric clock
2. Timing device
3. Tape decks
4. Turn tables and other precision equipments.